A Model of Sales

By Hal R. Varian*

Economists have belatedly come to recognize that the "law of one price" is no law at all. Most retail markets are instead characterized by a rather large degree of price dispersion. The challenge to economic theory is to describe how such price dispersion can persist in markets where at least some consumers behave in a rational manner. Starting with the seminal paper of George Stigler, a number of economic theorists have proposed models to describe this phenomenon of equilibrium price dispersion. See, for example, Gerard Butters, John Pratt, David Wise, and Richard Zeckhauser, Michael Rothschild, Steven Salop, Salop and Joseph Stiglitz (1977), Yuval Shilony, Stiglitz, and Louis Wilde and Alan Schwartz.

Most of the models of price dispersion referred to above are concerned with analyzing "spatial" price dispersion; that is, a situation where several stores contemporaneously offer an identical item at different prices. A nice example of such a model is the "bargains and ripoffs" paper of Salop and Stiglitz (1977). They consider a market with two kinds of consumers; the "informed" consumers know the entire distribution of offered prices, while the "uninformed" consumers know nothing about the distribution of prices. Hence the informed consumers always go to a low-priced store, while the uninformed consumers shop at random. The stores have identical U-shaped cost curves and behave as monopolistically competitive price setters. Salop and Stiglitz show that for some parameter configurations, the market equilibrium takes a form where some fraction of the stores sell at the competitive price (minimum average cost) and some fraction sell at a higher price. The high-price stores' clientele consists only of uninformed consumers, but there is a sufficiently large number of them to keep the stores in business.

In the Salop and Stiglitz model—as in all the models of spatial price dispersion—some stores are supposed to *persistently* sell their product at a lower price than other stores. If consumers can learn from experience, this persistence of price dispersion seems rather implausible.

An alternative type of price dispersion might be called "temporal" price dispersion. In a market exhibiting temporal price dispersion, we would see each store varying its price over time. At any moment, a cross section of the market would exhibit price dispersion; but because of the intentional fluctuations in price, consumers cannot learn by experience about stores that consistently have low prices, and hence price dispersion may be expected to persist.

One does not have to look far to find the real world analog of such behavior. It is common to observe retail markets where stores deliberately change their prices over time—that is, where stores have *sales*. A casual glance at the daily newspaper indicates that such behavior is very common. A high percentage of advertising seems to be directed at informing people of limited duration sales of food, clothing, and appliances.

Given the prevalence of sales as a form of retailing, it is surprising that so little attention has been paid to sales in the literature of economic theory. In fact, I know of no work in economic theory that *explicitly* examines the rationale of price dispersion by means of sales.[1] However, the work of Shilony does provide an implicit rationale for the use of sales as a marketing device.

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[1]Salop and Stiglitz' 1976 paper is concerned with "spatial" price dispersion rather than temporal price dispersion.

Shilony examines an oligopolistic market where consumers can purchase costlessly from neighborhood stores, but incur a "search cost" if they venture to more distant stores in search of a lower price. He shows that no Nash equilibrium exists in pure pricing strategies. On the other hand, Shilony does establish the existence of an equilibrium *mixed* strategy—that is, a strategy where firms randomize their prices. Such a strategy could be interpreted as stores having randomly chosen sales.

In this paper, I explicitly address the question of sales equilibria. The model may be regarded as a combination of the Salop-Stiglitz and the Shilony models described above. As in the Salop-Stiglitz model, it will be assumed that there are informed and uninformed consumers. As in the Shilony model, I will allow for the possibility of randomized pricing strategies by stores. I will be interested in characterizing the equilibrium behavior in such markets.

In the model to be described below, firms engage in sales behavior in an attempt to price discriminate between informed and uninformed customers. This is of course only one aspect of real world sales behavior. Other reasons for sales behavior might include inventory costs, cyclical fluctuations in costs or demand, loss leader behavior, advertising behavior, and so on. The theoretical examination of these motives is left for future work.

I. The Model

Let us suppose there is a large number of consumers who each desire to purchase, at most, one unit of some good. The maximum price any consumer will pay for the good—a consumer's reservation price—will be denoted by r. Consumers come in two types, informed and uninformed.[2] Uninformed consumers shop for the item by choosing a store at random; if the price of the item in that store is less than r, the consumer purchases it. Informed consumers, on the other hand, know the whole distribution of prices, and in particular they know the lowest available price at any time. Hence, they go to the store with the lowest price and purchase the item there.

One might think of a model where stores advertise their sale prices in the weekly newspaper. Informed consumers read the newspaper and uninformed consumers do not. Let $I>0$ be the number of informed consumers, and $M>0$ the number of uninformed consumers. Let n be the number of stores, and let $U=M/n$ be the number of uninformed consumers per store.

Each store has a density function $f(p)$ which indicates the probability with which it charges each price p. In its choice of this pricing strategy, each firm takes as given the pricing strategies chosen by the other firms and the demand behavior of the consumers. Only the case of a symmetric equilibrium will be examined, where each firm chooses the same pricing strategy.[3]

Each week, each store randomly chooses a price according to its density function $f(p)$. A store *succeeds* in its sale if it turns out to have the lowest price of the n prices being offered. In this case the store will get $I+U$ customers. If a store *fails* to have the lowest price, it will get only its share of uninformed customers, namely U. If two or more stores charge the lowest price, it will be considered a *tie*, and the low-price stores will each get an equal share of the informed customers.

Finally the stores are characterized by identical, strictly declining average cost curves.[4] The cost curve of a representative firm will be denoted by $c(q)$. It will be assumed that entry occurs until (expected) profits are driven to zero. Thus we will be examining a symmetric monopolistically competitive equilibrium in pricing strategies.

[2]For now, the uninformed-informed distinction is exogenously given. The decision to become informed or uninformed will be examined in Section III.

[3]Some justification for this symmetry assumption is given by Proposition 9 in the Appendix.

[4]The motivation for this assumption is the casual observation that retail stores are characterized by fixed costs of rent and sales force, plus constant variable costs—the wholesale cost—of the item being sold.

II. The Analysis

The maximum number of customers a store can get is $I+U$. Let $p^*=c(I+U)/(I+U)$ be the average cost associated with this number of customers.

PROPOSITION 1: *$f(p)=0$ for $p>r$ or $p<p^*$.*

PROOF:
No price above the reservation price will be charged since there is zero demand at any such price. No price less that p^* will be charged since only negative profits can result from such a price.

PROPOSITION 2: *There is no symmetric equilibrium where all stores charge the same price.*

PROOF:
Suppose that all stores were charging a single price p with $r\geqq p>p^*$. Then a slight cut in price by one of the stores would capture all of the informed market, and thus make a positive profit. If all stores were charging p^*, each would get an equal share of the market and thus be making negative profits.

Proposition 2 is simply a variant of the well-known argument that declining average cost curves and "competitive" behavior are incompatible. I therefore concentrate on establishing the nature of a price-randomizing solution. Recall that p is a *point mass* of a probability density function f if there is positive probability concentrated at p.

PROPOSITION 3. *There are no point masses in the equilibrium pricing strategies.*[5]

PROOF:
The intuition of this argument is seen to be quite straightforward. If some price p were charged with positive probability, there would be a positive probability of a tie at p. If a deviant store charged a slightly lower price, $p-\varepsilon$, with the same probability with which the other stores charged p, it would lose profits on order ε, but gain a fixed positive amount of profits when the other stores tied. Thus for small ε its profits would be positive, contradicting the assumption of equilibrium.

Let us proceed to a detailed formulation of this argument. First note that p^* can never be charged with positive probability, for when p^* is the lowest price charged, profits are zero, and if there is a tie at p^*, profits are negative. Suppose then that $p>p^*$ is charged with positive probability.

The number of points of positive mass in any probability distribution must be countable so we can find an arbitrarily small ε such that $p-\varepsilon$ is charged with probability 0. Consider what happens if we charge $p-\varepsilon$ with the probability with which we used to charge p, and charge p with probability 0. The increase in profits will be

$$
\begin{aligned}
&Pr(P_i>p-\varepsilon \text{ all } i,\ P_i\neq p \text{ any } i)\\
&\qquad\qquad((p-\varepsilon)(I+U)-c(I+U))\\
&\quad -Pr(P_i>p \text{ all } i)\ (p(I+U)-c(I+U))\\
&\quad +Pr(p_i<p-\varepsilon \text{ some } i)\quad ((p-\varepsilon)U-c(U))\\
&\qquad\qquad -Pr(P_i<p \text{ some } i)(pU-c(U))\\
&\quad +\sum_{k=2}^{n} Pr(P_i\geqq p-\varepsilon \text{ all } i,\ P_i=p \text{ for } k \text{ stores})\\
&\qquad\qquad((p-\varepsilon)(I+U)-c(I+U))\\
&\quad -\sum_{k=2}^{n} Pr(P_i\geqq p \text{ all } i,\ P_i=p \text{ for } k \text{ stores})\\
&\qquad\qquad(p(U+I/k)-c(U+I/k))
\end{aligned}
$$

As ε approaches zero, the sum of the first four terms approaches zero, while the sum of the last two terms remains a positive number. Hence for small ε profits are positive, contradicting the assumption of an equilibrium strategy.

Proposition 3 expresses the essential difference between models of spatial price

[5]Proposition 9 in the Appendix provides a partial converse to this assertion.

dispersion and models of temporal price dispersion. Most models of spatial price dispersion, such as the Salop-Stiglitz model or the Wilde-Schwartz model, have equilibria with specific prices being charged with positive probability mass. The above argument shows that such strategies cannot be profit-maximizing Nash behavior in a temporal randomizing model.

Since there are no point masses in the equilibrium density, the cumulative distribution function will be a continuous function on (p^*, r). Let $F(p)$ be the cumulative distribution function for $f(p)$; thus $f(p) = F'(p)$ almost everywhere.

We can now construct the expected profit function for a representative store. When a store charges price p, exactly two events are relevant. It may be that p is the smallest price being charged, in which case, the given store gets all of the informed customers. This event happens only if all the other stores charge prices higher than p, an event which has probability $(1-F(p))^{n-1}$. On the other hand, there may be some store with a lower price, in which case the store in question only gets its share of the uninformed customers. This event happens with probability $1-(1-F(p))^{n-1}$. (By Proposition 3 we can neglect the probability of any ties.) Hence the expected profit of a representative store is

$$\int_{p^*}^{r}\Big\{\pi_s(p)(1-F(p))^{n-1} + \pi_f(p)\big[1-(1-F(p))^{n-1}\big]\Big\} f(p)\,dp$$

where $\pi_s(p) = p(U+I) - c(U+I)$

$$\pi_f(p) = pU - c(U)$$

The maximization problem of the firm is to choose the density function $f(p)$ so as to maximize expected profits subject to the constraints:

$$f(p) \geqq 0; \quad \int_{p^*}^{r} f(p)\,dp = 1$$

It is clear that all prices that are charged with positive density must yield the same expected profit; for if some price yields a greater profit than some other price it would pay to increase the frequency with which the more profitable price were charged. Since we require zero profits due to free entry, this common level of profit must be zero.[6] This argument yields

PROPOSITION 4: *If $f(p) > 0$, then*

$$\pi_s(p)(1-F(p))^{n-1} + \pi_f(p)\big[1-(1-F(p))^{n-1}\big] = 0$$

(Of course, Proposition 4 also follows directly from the application of the Kuhn-Tucker theorem to the specified maximization problem.) Rearranging this equation, we have a formula for the equilibrium cumulative distribution function:

$$1 - F(p) = \left(\frac{\pi_f(p)}{\pi_f(p) - \pi_s(p)}\right)^{\frac{1}{n-1}}$$

Note that the denominator of this fraction is negative for any p between p^* and r. Hence the numerator must be negative so that profits in the event of failure are definitely negative. The construction of $(1-F(p))^{n-1}$ is illustrated in Figure 1. At each p where $f(p) > 0$ we can construct $\pi_f(p)$ and $\pi_s(p)$ as illustrated and take the relevant ratio. Proposition 4 gives us an explicit expression for the equilibrium distribution function at those values of p where $f(p) > 0$. If this is to be a legitimate candidate for a cumulative distribution function, it should be an increasing function of p. This is easy to verify:

PROPOSITION 5: $\pi_f(p)/(\pi_f(p) - \pi_s(p))$ *is strictly decreasing in p.*

PROOF:

Taking the derivative it suffices to show that

$$(\pi_f(p) - \pi_s(p))U - \pi_f(p)(-I) < 0$$

[6] One can also formulate the model with a fixed number of firms. In this case, expected profits must be equal to $\Pi_f(r)$.

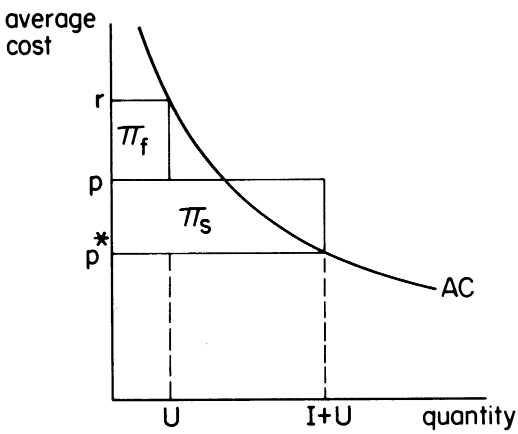

FIGURE 1. GRAPHICAL DETERMINATION OF $\pi_s(p)$ AND $\pi_f(p)$

Using the definitions of π_f and π_s, this can be rearranged to yield

$$\frac{c(I+U)}{I+U} < \frac{c(U)}{U}$$

which is obvious since average cost has been assumed to strictly decrease.

Of course, Proposition 4 characterizes the equilibrium density function only for those prices where $f(p)>0$. In order to fully characterize the equilibrium behavior, we need to establish which prices are charged with positive density.

First, it is clear that prices close to p^* must be charged with positive density:

PROPOSITION 6: *$F(p^*+\varepsilon)>0$ for any $\varepsilon>0$.*

PROOF:

If not, some store could charge $p^*+\varepsilon/2$, and thereby undercut the rest of the market and make positive profits.

Similarly we can characterize the behavior of $f(p)$ near its upper limit.

PROPOSITION 7: *$F(r-\varepsilon)<1$ for any $\varepsilon>0$.*

PROOF:[7]

Suppose not, and let $\hat{p}<r$ be the highest price that is ever charged so that $F(\hat{p})=1$. When $\hat{p}$ is charged, the store will only get the uninformed customers since with probability 1 some other store will be charging a lower price. Since the store must get zero expected profits at each price charged, we must have $\hat{p}U-c(U)=0$. But then $rU-c(U)>0$, so charging r with probability 1 could make a positive profit.

Propositions 6 and 7 show that prices near p^* and r are charged with positive density. It is now easy to show:

PROPOSITION 8: *There is no gap (p_1, p_2) where $f(p)\equiv 0$.*

PROOF:

If not, let $p_1<\hat{p}<p_2$. Now $\hat{p}$ succeeds in being the lowest price in exactly the same circumstances that p_1 succeeds in being the lowest price; namely, when all other prices are greater than p_2. Similarly, $\hat{p}$ fails to be the lowest price when some store charges a price less than p_1, in which case p_1 also fails to be the lowest price. But in each circumstance, since $\hat{p}>p_1$, $\hat{p}$ will make larger profits than p_1. Since p_1 must make zero profits, this shows that charging $\hat{p}$ with probability 1 will make positive profits.

We now have a complete characterization of the equilibrium density: $f(p)>0$ for all p in (p^*, r) and $f(p)=F'(p)$, where

$$(1) \qquad F(p)=1-\left(\frac{\pi_f(p)}{\pi_f(p)-\pi_s(p)}\right)^{\frac{1}{n-1}}$$

We can also solve for the endogenous variables n and p^*. First, note that if a store charges r, it only gets the uninformed customers, and profits must therefore satisfy $\pi_f(r)=0$. Similarly, if a store charges p^* it gets all the informed customers with probability 1 so $\pi_s(p^*)=0$. These two equations can be used to determine n and p^*.

[7]A heuristic proof is presented here and a more rigorous proof in the Appendix. (The same holds true for Proposition 8.)

As an example, let us compute the equilibrium density when the cost function has fixed cost $k>0$ and zero marginal cost. Then

$$\pi_s(p)=p(I+U)-k \tag{2}$$

$$\pi_f(p)=pU-k \tag{3}$$

Since $\pi_f(r)=0$, and $U=M/n$

$$rM/n-k=0 \tag{4}$$

or

$$n=rM/k \tag{5}$$

Thus

$$U=M/n=k/r \tag{6}$$

Since $\pi_s(p^*)=0$, we have

$$p^*\left(I+\frac{k}{r}\right)-k=0$$

or

$$p^*=\frac{k}{I+k/r} \tag{7}$$

The equilibrium distribution function can be found by substituting (2) and (3) into (1). We have

$$F(p)=1-\left(\frac{k-pU}{pI}\right)^{\frac{1}{n-1}} \tag{8}$$

Substituting from (6) and rearranging, we find

$$F(p)=1-[(k/I)(1/p-1/r)]^{\frac{1}{n-1}} \tag{9}$$

The equilibrium density function is found by differentiating (9):

$$f(p)=F'(p) \tag{10}$$

$$=\frac{(k/I)^{\frac{1}{n-1}}}{n-1}\frac{(1/p-1/r)^{\frac{1}{n-1}-1}}{p^2}$$

Let

$$m=1-\frac{1}{n-1}=\frac{n-2}{n-1}=\frac{rM-2k}{rM-k} \tag{11}$$

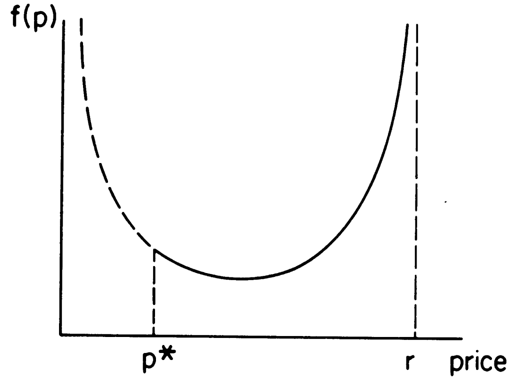

FIGURE 2. GRAPH OF $f(p)=1/p(1-p/r)$

Then $f(p)$ can be written as

$$f(p)=\frac{k(k/I)^{1-m}}{(rM-k)}\frac{1}{p^{2-m}(1-p/r)^m} \tag{12}$$

If n is reasonably large, m will be approximately 1, so $f(p)$ will be proportional to

$$\frac{1}{p(1-p/r)} \tag{13}$$

This density is illustrated in Figure 2. Note that stores tend to charge extreme prices with higher probability than they charge intermediate prices. This seems intuitively plausible; a store would like to discriminate in its pricing and charge informed customers p^* (to keep their business) and charge uninformed customers r (to exploit their surplus). Since they are required to sell to all consumers at the same price this tendency shows up in the U-shaped density of prices.

The relevant part of the density is of course that part between p^* and r. Referring to equation (7) we see that if fixed costs are small, or the number of informed consumers is large, p^* will be small. Hence low prices will be charged a high percentage of the time. In some sense the market will be more competitive. On the other hand, the influence of the uninformed consumers is never entirely absent, since high prices will always be charged for some fraction of the time.

TABLE 1—SUMMARY OF COMPARATIVE STATICS

	k	I	M	r	λ
p^*	+	−	0	+	−
n	−	0	+	+	−
$F(p)$	?	+	−	−	+
$\bar{p}$	?	−	+	+	−
$\bar{p}_{min}$	?	?	−	?	?

Note: K=fixed costs, I=number of informed consumers, M=number of uninformed consumers, r=reservation prices, and λ=fraction of informed consumers: $\lambda = I/(I+M)$.

It is of interest to calculate the average price paid by the informed and the uninformed consumers under the equilibrium price density. The average price the uninformed consumers pay is simply:

$$\bar{p} = \int_{p^*}^{r} pf(p)\,dp$$

If we integrate this by parts we have

$$\bar{p} = r - \int_{p^*}^{r} F(p)\,dp$$

Substituting from (9)

$$\bar{p} = p^* + \left(\frac{k}{I}\right)^{\frac{1}{n-1}} \int_{p^*}^{r} \left(\frac{1}{p} - \frac{1}{r}\right)^{\frac{1}{n-1}} dp \tag{14}$$

There seems to be no simple expression for the integral in (14). However in the duopoly case (where $n=2$) the expression becomes rather trivial. Suppose for example that we have $r=1$, $M=2$, $I=1$, $k=1$. Then by (5) we have $n=2$, and by (7), $p^*=.5$. Substituting into (14),

$$\bar{p} = .5 \int_{.5}^{1} \left(\frac{1}{p} - 1\right) dp = -\ln .5 \simeq .69 \tag{15}$$

Let us denote the price paid by the informed customers by p_{min}. Then the density function of p_{min} is just

$$f_{min}(p) = (1-F(p))^{n-1} f(p)$$

Substituting from (9)

$$f_{min}(p) = \frac{k}{I}(1/p - 1/r) f(p)$$

Thus, $$\bar{p}_{min} = \frac{k}{I}\int_{p^*}^{r} p(1/p - 1/r) f(p)\,dp$$

$$= \frac{k}{I}\left(1 - \frac{\bar{p}}{r}\right) \tag{16}$$

For the example given above, $\bar{p}_{min} \simeq .31$.

Table 1 presents the results of the comparative statics computations. The signs are for the most part as expected. However, the behavior of $\bar{p}$ with respect to M—the number of uninformed consumers—does contain an interesting feature. It can be shown that $\partial \bar{p}/\partial M > 0$. That is, more uninformed consumers cause the average price paid by the uninformed consumers to rise. This is an example of the detrimental externalities that noneconomizing behavior can impose. However, note from (16) that $\bar{p}_{min}$ will *decrease* with M—the uninformed consumers confer a *beneficial* externality on the informed consumers. This effect seems to arise because the number of stores is increasing in the number of uninformed consumers; since the informed consumers buy at the *lowest* advertised price, a larger number of stores tends to lower the average price they pay.

III. Does it Pay to be Informed?

The model presented above takes the informed and uninformed consumers as exogenously given. However, the decision to become informed or uninformed can easily be made endogenous. Following the Salop-Stiglitz example, let us now suppose that it is possible to become fully informed about the available prices in the market by paying a fixed cost c. We think of this as the cost involved in reading newspaper advertisements, processing the information, and so on. Further, suppose that there are two types of consumers: one group has "search costs" c_2 and the other has search costs c_1, with $c_2 \geqq c_1$.

The decision to be informed or uninformed now depends on the "full price" one pays to purchase the product in question. An uninformed person pays $\bar{p}$ on the average while an informed person pays $\bar{p}_{min}+c_i$, $i=1,2$. In order for the equilibrium to be a full equilibrium, neither the informed, nor the uninformed group should find it in their interest to change their behavior.

For example, suppose $c_2 > c_1 = 0$. Then the low-cost consumers will always be informed. If the high-cost customers also find it in their interest to be informed no equilibrium will exist.[8] It is in the interest of the high-cost consumers to remain uninformed if $\bar{p} < \bar{p}_{min} + c_2$. Using the results of (15) and (16), this reduces to $\bar{p} < k(1-\bar{p}/r)/I + c_2$, or $\bar{p}/r < (k+c_2 I)/(k+rI)$. If c_2 is greater than r, for example, this condition will certainly be satisfied.

IV. Summary

I have shown how stores may find it in their interest to randomize prices in an attempt to price discriminate between informed and uninformed consumers, and have solved explicitly for the resulting monopolistically competitive equilibrium in randomized pricing strategies. The form of the resulting pricing strategy as given in Figure 2 does not seem out of line with commonly observed retailing behavior. Large retailing chains such as Sears and Roebuck and Montgomery Ward sell appliances at their regular price much of the time, but often have sales when the price is reduced by as much as 25 percent. However, we rarely observe them selling an appliance at an intermediate price. Although this casual empiricism can hardly be conclusive, it suggests that the features of the model described here may have some relevance in explaining real world retailing behavior.[9]

Appendix

PROPOSITION 7: *$F(r-\varepsilon) < 1$ for any $\varepsilon > 0$.*

PROOF:

If not, let $\hat{p} < r$ satisfy $F(\hat{p}) = 1$ and let (p^i) be a sequence of prices with $f(p^i) > 0$ and $(p^i) \to \hat{p}$. Clearly $F(p^i) \to 1$, so

$$1 = 1 - \left(\frac{\pi_f(\hat{p})}{\pi_f(\hat{p}) - \pi_s(\hat{p})} \right)^{\frac{1}{n-1}}$$

Hence $\pi_f(\hat{p}) = 0$, or $\hat{p}U - c(U) = 0$. But then $rU - c(U) > 0$, so a store charging r with probability 1 would make a positive profit.

PROPOSITION 8: *There is no gap (p_1, p_2) where $f(p) \equiv 0$.*

PROOF:

If not, let (p_1, p_2) be the largest such gap and let (p^i) and (p^j) be sequences of prices in the support of f converging to p_1 and p_2, respectively. Then, $\lim F(p^i) = \lim F(p^j)$ since F is continuous, which implies

$$\frac{\pi_f(p_1)}{\pi_f(p_1) - \pi_s(p_1)} = \frac{\pi_f(p_2)}{\pi_f(p_2) - \pi_s(p_2)}$$

According to Proposition 5 this is impossible unless $p_1 = p_2$.

PROPOSITION 9: *If each store's optimal strategy involves zero probability of a tie, and $f(p) > 0$ for all $p^* \leqq p < r$, then each store must choose the same strategy.*

PROOF:

Let $F_i(p)$ be the optimal strategy for store $i = 1, \ldots, n$. Then by the reasoning of Proposition 4, stores k and j must satisfy the

[8] If all consumers are informed, only the lowest-price store will have any customers, and there can be only one price charged in equilibrium. But this is impossible by Proposition 2.

[9] More detailed empirical data on sales behavior of appliance retailers is presented in my working paper.

equations

$$\pi_s(p)\prod_{i\neq j}(1-F_i(p))^{n-1}$$

$$=-\pi_f(p)\prod_{i\neq j}\left[1-(1-F_i(p))^{n-1}\right]$$

$$\pi_s(p)\prod_{i\neq k}(1-F_i(p))^{n-1}$$

$$=-\pi_f(p)\prod_{i\neq k}\left[1-(1-F_i(p))^{n-1}\right]$$

Dividing one equation into the other, we have

$$\frac{(1-F_k(p))^{n-1}}{(1-F_j(p))^{n-1}}=\frac{1-(1-F_k(p))^{n-1}}{1-(1-F_j(p))^{n-1}}$$

which implies $F_j(p)=F_k(p)$.

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