Revealed Preference

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Abstract

This is a survey of revealed preference analysis focusing on the period since Samuelson's seminal development of the topic with emphasis on empirical applications. It was prepared for *Samuelsonian Economics and the 21st Century*, edited by Michael Szenberg.

1 Introduction

In January 2005 I conducted a search of JSTOR business and economics journals for the phrase "revealed preference" and found 997 articles. A search of Google scholar returned 3,600 works that contained the same phrase. Surely, revealed preference must count as one of the most influential ideas in economics. At the time of its introduction it was a major contribution to the pure theory of consumer behavior, and the basic idea has been applied in a number of other areas of economics.

In this essay I will briefly describe of the history of revealed preference, starting first descriptions of the concept in Samuelson's papers. These papers subsequently stimulated a substantial amount of work devoted to refinements

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and extension of Samuelson's ideas. This theoretical works, in turn, led to a literature on the use of revealed preference analysis for empirical work that is still growing rapidly.

2 The pure theory of revealed preference

Samuelson [1938] contains the first description of the concept he later called "revealed preference." The initial terminology was "selected over."[1]

In this paper, Samuelson stated what has since become known as the "Weak Axiom of Revealed Preference" by saying "...if an individual selects batch one over batch two, he does not at the same time select two over one." Let us state Samuelson's definitions a bit more formally.

Definition 1 (Revealed Preference) *Given some vectors of prices and chosen bundles* (p^t, x^t) *for* $t = 1, \ldots, T$*, we say* x^t *is directly revealed preferred to a bundle* x *(written* $x^t R_D x$*) if* $p^t x^t \geq p^t x$*. We say* x^t *is revealed preferred to* x *(written* $x^t R x$*) if there is some sequence* $r, s, t, \ldots, u, v$ *such that* $p^r x^r \geq p^r x^s$*,* $p^s x^s \geq p^s x^t, \cdots, p^u x^u \geq p^u x$*. In this case, we say the relation* R *is the transitive closure of the relation* R_D*.*

Definition 2 (Weak Axiom of Revealed Preference) *If* $x^t R_D x^s$ *then it is not the case that* $x^s R_D x^t$*. Algebraically,* $p^t x^t \geq p^t x^s$ *implies* $p^s x^s < p^s x^t$*.*

Subsequently, building on the work of Little [1949], Samuelson [1948] sketched out an argument describing how one could use the revealed preference relation to construct a set of indifference curves. This proof was for two goods only, and was primarily graphical. Samuelson recognized that a general proof for multiple goods was necessary, and left this as an open question.

[1]As Richter [1966] has pointed out, "selected over" has the advantage over "revealed preference" in that it avoids confusion about circular definition of "preference." Unfortunately, the original terminology didn't catch on.

Houthakker [1950] provided the missing proof for the general case. As Samuelson [1950] put it, "He has given us the long-sought test for integrability that can be formed in finite index-number terms, without need to estimate partial derivatives."

Houthakker's contribution was to recognize that one needed to extend the "direct" revealed preference relation to what he called the "indirect" revealed preference relation or, for simplicity, what we call the "revealed preference" relation. Houthakker's condition can be stated as:

Definition 3 (Strong Axiom of Revealed Preference (SARP)) *If $x^t R x^s$ then it is not the case that $x^s R x^t$. Algebraically, SARP says $x^t R x^s$ implies $p^s x^s < p^s x^t$.*

Rose [1958] later offered a formal argument that the Strong Axiom and the Weak Axiom were equivalent in two dimensions, providing a rigorous, algebraic foundation for Samuelson's earlier graphical exposition. (See Afriat [1965] for a different proof.)

Samuelson [1953], stimulated by Hicks [1939], summed up all of consumer theory in what he called the *Fundamental Theorem of Consumption Theory.* "Any good (simple or composite) that is known always to increase in demand when money income alone rises must definitely shrink in demand when its price alone rises." In this paper he lays out a graphical and algebraic description of the Slutsky equation and the restrictions imposed by consumer optimization. Yokoyama [1968] elegantly combined Samuelson's verbal and algebraic treatments of the Slutsky equation and made the connection between the Samuelson and the Hicks approaches explicit.

By 1953 the basic theory of consumer behavior in terms of revealed preference was pretty much in place, though it was not completely rigorous. Subsequent contributions, such as Newman [1960], Uzawa [1960], and Stigum [1973] added increasing rigor to Houthakker and Samuelson's arguments.

During the same period Richter [1966] recognized that one could dispense with the traditional integrability approach using differential equations and base revealed preference on pure set-theoretic arguments involving the completion of partial orders.

This period culminated in the publication of Chipman et al. [1971], which contained a series of chapters that would seem to be the last word on revealed preference. Several years later Sondermann [1982], following Richter [1966]'s analysis, provided a one-paragraph proof of the basic revealed preference result, albeit a proof that used relatively sophisticated mathematics.

3 Afriat's approach

Most of the theoretical work described above starts with a demand function: a complete description of what would be chosen at any possible budget. Afriat [1967] offered quite a different approach to revealed preference theory. He started with a *finite* set of observed prices and choices and asked how to actually construct a utility function that would be consistent with these choices.[2]

The standard approach showed, in principle, how to construct preferences consistent with choices, but the actual preferences were described as limits or as a solution to some set of partial differential equations.

Afriat's approach, by contrast, was truly constructive, offering an explicit algorithm for to calculate a utility function consistent with the finite amount of data, whereas the other arguments were just existence proofs. This makes Afriat's approach much more suitable as a basis for empirical analysis.

Afriat's approach was so novel that most researchers at the time did not recognize its value. In addition, Afriat's exposition was not entirely transparent. Several years later Diewert [1973] offered a somewhat clearer exposition of Afriat's main results.

[2] I once asked Samuelson whether he thought of revealed preference theory in terms of a finite or infinite set of choices. His answer, as I recall, was: "I thought of having a finite set of observations . . . but I always could get more if I needed them!"

4 From theory to data

During the late 1970s and early 1980s there was considerable interest in estimating aggregate consumer demand functions. Christensen, Lars, Jorgensen and Lau [1975] and Deaton [1983] are two notable examples. In reading this work, it occurred to me that it could be helpful to use revealed preference as a pre-test for this econometric analysis.

After all, the Strong Axiom of Revealed Preference was a necessary and sufficient condition for data to be consistent with utility maximization. If the data satisfied SARP there would be *some* utility function consistent with the observations. If the data violated SARP, no such utility function would exist. So why not test those inequalities directly?

I dug into the literature a bit and discovered that Koo [1963] had already thought of doing this, albeit with a somewhat different motivation. However, as Dobell [1965] pointed out, his analysis was not quite correct so there was still something left to be done.

Furthermore I recognized the received theory, using WARP and SARP, was not well-suited for empirical work, since it was built around the assumption of single-valued demand functions. In 1977, during a visit to Berkeley, Andreu Mas-Collel pointed me to Diewert [1973]'s exposition of Afriat's analysis, which seemed to me to be a more promising basis for empirical applications.

Diewert [1973] in turn led to Afriat [1967]. I corresponded with Afriat during this period, and he was kind enough to send me a package of his writing on the subject. His monograph Afriat [1987] offered the clearest exposition of his work in this area, though, as I discovered, it was not quite explicit enough to be programmed into a computer.

I worked on reformulating Afriat's argument in a way that would be directly amenable to computer analysis. While doing this, I recognized that Afriat's condition of "cyclical consistency" was basically equivalent to Strong Axiom. Of course, in retrospect this had to be true since both cyclical consistency and SARP were necessary and sufficient conditions for utility maximization. Even though the proof was quite straightforward, this was a big help to my understanding since it pulled together the quite different approaches of Afriat and Houthakker.

During 1978-79 I worked on writing a program for empirical revealed preference analysis. The code was written in FORTRAN77 and ran on the University of Michigan MTS operating system on an IBM mainframe. This made is rather unportable, but then again this was before the days of personal computers, so everything was unportable. During 1980-81 I was on leave at Nuffield College, Oxford and became more and more intrigued by the *empirical* applications of revealed preference. As I saw it, the main empirical questions could be formulated in the following way.

Given a set of observations of prices and chosen bundles, (p^t, x^t) for $t = 1, \ldots T$ we can ask four basic questions.

Consistency. When is the observed behavior consistent with utility maximization?

Form. When is the observed behavior consistent with maximizing a utility function of particular form?

Recoverability. How can we recover the set of utility functions that are consistent with a given set of choices?

Forecasting. How can we forecast what demand will be at some new budget?

In the rest of the paper I will review some of the literature concerned with pursuing answers to these four basic questions.

5 Consistency

Consistency is, of course, the central focus of the early work on revealed preference. As we have seen, several authors contributed to its solution, including Samuelson, Houthakker, Afriat and others. The most convenient result for empirical work, as I suggested above, comes from Afriat's approach.

Definition 4 (Generalized Axiom of Revealed Preference) *The data* (p^t, x^t) *satisfy the Generalized Axiom of Revealed Preference (GARP) if* $x^t R x^s$ *implies* $p^s x^s \leq p^s x^t$*.*

GARP, as mentioned above, is equivalent to what Afriat called "cyclical consistency." That the only difference between GARP and SARP is that the strong inequality in SARP becomes a weak inequality in GARP. This allows for multivalued demand functions and "flat" indifference curves, which turns out to be important in empirical work.

Now we can state the main result.

Theorem 1 (Afriat's Theorem.) *Given some choice data* (p^t, x^t) *for* $t = 1, \ldots, T$*, the following conditions are equivalent.*

1. *There exists a nonsatiated utility function* $u(x)$ *that rationalizes the data in the sense that for all* t*,* $u(x^t) \geq u(x)$ *for all* x *such that* $p^t x^t \geq p^t x$*.*

2. *The data satisfy GARP.*

3. *There is a positive solution* (u^t, λ^t) *to the set of linear inequalities*

$$u^t \leq u^s + \lambda^s p^s (x^t - x^s) textforalls, t.$$

4. *There exists a nonsatiated, continuous, monotone, and concave utility function* $u(x)$ *that rationalizes the data.*

This theorem offers two equivalent, testable conditions for the data to be consistent with utility maximization. The first is GARP, which, as we have seen, is a small generalization of Houthakker's SARP. The second condition is whether there is a positive solution to a certain set of linear inequalities. This can easily be checked by linear programming methods. However, from the viewpoint of computational efficiency it is much easier just to check GARP. The only issue is to figure out how to compute the revealed preference relation in an efficient way.

Let us define a matrix m that summarizes the direct revealed preference relation. In this matrix the (s, t) entry is given by $m_{st} = 1$ if $p^t x^t \geq p^t x^s$ and $m_{st} = 0$ otherwise. In order to test GARP all that is necessary is to compute the transitive closure of the relation summarized by this matrix. What algorithms are appropriate?

Dobell [1965] recognized that this could be accomplished simply by taking the Tth power of the $T \times T$ binary matrix that summarizes the direct revealed preference relation. However, it turned out the computer scientists had a much more efficient algorithm. Warshall [1962] had shown a few years earlier how to use dynamic programming to compute the transitive closure in just T^3 steps.

Combining the work of Afriat and Warshall effectively solved the problem of finding a computationally efficient method of testing data for consistency with utility maximization. One could simply construct the matrix summarizing the direct relation, compute the transitive closure and then check GARP.

5.1 Empirical analysis

Several authors have tested revealed preference conditions on different sorts of data. The "best" data, in some sense, is experimental data involving individual subjects since one can vary prices in such a setting and so test choice behavior over a wide range of environments.

Battalio et al. [1873] was, I believe, the first paper to look at individual human subjects. The subjects were patients in a mental institution who were offered payments for good behavior. Cox [1989] later examined the same data and extended the analysis in several ways.

Kagel et al. [1995] summarizes several studies examining animal behavior. Harbaugh et al. [2001] examined choice behavior by children and Andreoni and Miller [2002] looked at public goods experiments to test for rational behavior in this context.

Individual household consumption data is the next best set of data to examine in the context of consumer choice theory. I believe that Koo [1963] was the first paper to look at household data. See also the subsequent exchange between Dobell [1965] and Koo [1965]. Later studies using household budgets include Manser and McDonald [1988] and Famulari [1995]. Dowrick and Quiggin [1994] and Dowrick and Quiggin [1997]look at international aggregate data.

Finally, we have time series data on aggregate consumption. I used these methods described above to test revealed preference in Varian [1982a]. To my surprise, the aggregate consumption data easily satisfied the revealed preference conditions. I soon realized that this was for a trivial reason: the changes expenditure from year to year were large relative to the changes in relative prices. Hence budget sets rarely intercepted in ways that would generate a GARP violation (or so it seemed).

Bronars [1985] offered a novel contribution by investigating the power of the GARP test. Power, of course, can only be measured against a specific alternative hypothesis, and Bronars chose the Becker [1962] hypothesis of random choice on the budget set. He found that Becker's random choice model violated GARP about 67 percent of the time. Contrary to my original impression, there were apparently enough budget intersections in aggregate time series to give GARP some bite.

GARP was even more powerful on per capita data. Of course, another interpretation of these findings is that Becker's random choice model isn't a very appealing alternative hypothesis. But, for all the criticism directed at the classical theory of consumer behavior there seem to be few alternative hypotheses other than Becker's that can be applied using the same sorts of data used for revealed preference analysis.

5.2 Goodness of fit

It is of interest to consider ways to relax the revealed preference tests so that one might say "these data are *almost* consistent with GARP." Afriat [1967] defines a "partial efficiency" measure which can be used to measure how well a given set of data satisfies utility maximization.

Definition 5 (Efficiency levels) *We say that x^t is directly revealed preferred to x at efficiency level e if $ep^tx^t \geq p^tx$.*

We define the transitive closure of this relation as R_e in the usual way. If $e = 1$ this is the standard direct revealed preference relation. If $e = 0$ nothing is directly revealed preferred to anything else, so GARP is vacuously satisfied. Hence there is some critical level e^* where the data just satisfy GARP.

It is easy to find the critical level e by doing a binary search. Varian [1990] suggests defining e^t separately for each observation and then finding those e^t that are as close as possible to 1 (in some norm). I interpret these e^t as a "minimal perturbation." They can be interpreted as error terms and thus be used to give a statistical interpretation to the goodness-of-fit measure.

Whitney and Swofford [1987] suggest using the number of violations as a fit measure, while Famulari [1995] uses a measure which is roughly the fraction of violations that occur divided by the fraction that could have occurred. Houtman and Maks [1985] proposes computing the maximal subset of the data that is consistent with revealed preference. These measures are reviewed and compared in Gross [1995] who also offers his own suggestions.

6 Form

The issue of testing for various sorts of separability had been considered by Afriat in unpublished work and independently examined by Diewert and Parkan [1985]. The Diewert-Parkhan work extended the linear inequalities described in Afriat's Theorem. They showed that if an appropriate set of linear inequalities had positive solutions, then the data satisfied the appropriate form restriction.

To get the flavor of this analysis, suppose that some observed data (p^t, x^t) were generated by a differentiable concave utility function $u(x)$. Differentiability and concavity imply that

$$u(x^t) \leq u(x^s) + Du(x^t)(x^s - x^t) \text{ for all } s, t$$

The first-order conditions for utility maximization imply

$$Du(x^t) = \lambda^t p^t \text{ for all } t.$$

Putting these together, we find that a *necessary* condition for the data to be consistent with utility maximization is that there is a set of positive numbers (u^t, λ^t), which can be interpreted as utility levels and marginal utilities of income, that satisfy the linear inequalities

$$u^t \leq u^s + \lambda^s(p^s x^t - p^t x^t) \text{ for all } s, t.$$

Furthermore, the existence of a solution to this set of inequalities is a *sufficient* condition as well. This can be proved by defining a utility function as the lower envelope of a set of hyperplanes defined as follows:

$$u(x) = \min_s u^s + \lambda^s p^s (x - x^s).$$

Afriat [1967] had used a similar construction but went further showed that cyclical consistency (i.e., GARP) was a necessary and sufficient condition for a solution to this set of linear inequalities to exist. Thus the computationally demanding task of verifying that a positive solution to a set of T^2 linear inequalities could be replaced by a much simpler calculation: checking GARP.

Suppose now that the data were generated by a homothetic utility function. Then it is well known that the indirect utility function can be represented as a multiplicatively separable function of price and income: $v(p)m$. This means that the marginal utility of income is simply $v(p)$, which also equals the utility level at income 1.

So if we normalize the observed prices so that expenditure equals 1 at each observation, we can write the above inequalities as

$$u^t \leq u^s + u^s(1 - p^s x^t) \text{ for all } s, t.$$

We have shown that the existence of a positive solution to these inequalities is a necessary condition for the maximization of a homothetic utility function. This can also be shown to be sufficient.

One immediately asks: is there an easier-to-check combinatorial condition that is equivalent to the existence of a solution for these inequalities. Varian [1982b] found such a condition. Simultaneously, Afriat [1981] published essentially the same test.

To get some intuition, consider Figure 1. The data (p^1, x^1) and (p^2, x^2) are consistent with revealed preference. However, if the underlying preferences are homothetic, then x^3 would be demanded at the budget set p^3, creating a violation of revealed preference.

In general, the necessary and sufficient condition for an observed set of choices to be consistent with homotheticity is given by HARP:

Definition 6 (Homothetic Axiom of Revealed Preference.) *A set of data (p^t, x^t) for $t = 1, \ldots, T$ satisfy the Homothetic Axiom of Revealed Pref-*

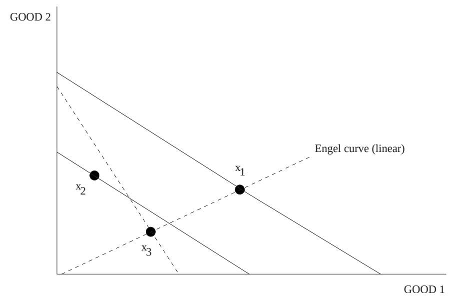

Figure 1: GARP with homothetic preference.

erence (HARP) if for every sequence $r, s, t, \ldots, u, v$

$$\frac{p^r x^s}{p^r x^r} \frac{p^s x^t}{p^s x^s} \cdots \frac{p^u x^v}{p^u x^u} \geq 1.$$

It turns out that there is an easy computation to check whether or not this condition is satisfied that uses methods that are basically the same as those in Warshall's algorithm.

Using similar methods, Browning [1984] came up with a nice test for life-cycle consumption models which rests on the constancy of the marginal utility of income in this framework.

Subsequently Blundell et al. [2003] recognized that the logic used in the homotheticity tests could be extended to a much more general setting.

Suppose one had estimates of Engel curves from other data. Then these Engel curves could be used to construct a set of data that could be subjected to revealed preference tests. The logic is the same as that described in Figure 1, but uses a an estimated Engel curve rather than the linear Engel curve implied by homotheticity. See Figure 2 for a simple example.

The Blundell-Browning-Crawford approach is very useful for empirical work since cross-sectional household data can be used to estimate Engel curves, either parametrically or nonparametrically. See Blundell [2005] for further developments in this area.

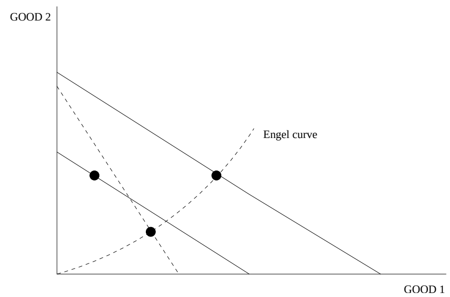

Figure 2: GARP with arbitrary Engel curve.

Other restrictions on functional form, such as various forms of separability, have been examined by Varian [1982a]. Tests for expected utility maximization and related models are described in Green and Srivastava [1986], Osbandi and Green [1991], Varian [1983], Varian [1988], Bar-Shira [1992].

7 Forecasting

Suppose we are given a finite set of observed budgets and choices (p^t, x^t) for $t = 1, \ldots, T$ that are consistent with GARP and a new price p^0 and expenditure y^0. What are the possible bundles x^0 that could be demanded at (p^0, y^0)?

Clearly all that is necessary is to describe the set of x^0 for which the (expanded) data set (p^t, x^t) for $t = 0, \ldots, T$ satisfy GARP. Varian [1982a] calls this the set of *supporting bundles.* Figure 3 shows the geometry.

In an analogous way, one can choose a new bundle x^0 and ask for the set of *prices* at which this bundle could be demanded. This is the set of *supporting prices.* Formally,

$$S(x^0) = \{p^0 : (p^t, x^t) \text{ satisfy GARP for } t = 0, \ldots T\}$$

Of course, one could also ask about demanded bundles or prices that are

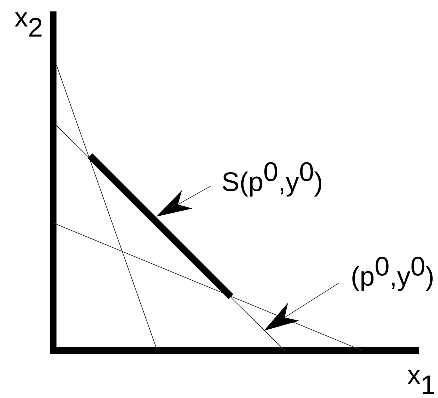

Figure 3: Supporting bundles.

consistent with utility functions with various restrictions imposed such as homotheticity, separability, specific forms for Engel curves and so on.

8 Recoverability

As we have seen, Afriat's methods can be used to construct a utility function that is consistent with finite set of observed choices that satisfy GARP. However, this is only one utility function. Typically there will be many other such functions. Is there a way to describe the entire set of utility functions (or preferences) consistent with some data?

Varian [1982a] posed the question in the following way. Suppose we are given a finite set of data (p^t, x^t) for $t = 2, \ldots T$ that satisfies GARP and two new bundles x^0 and x^1. Consider the set of prices at which which x^0 could be demanded, i.e., the supporting set of prices. If every such supporting set makes x^0 revealed preferred to x^1 then we conclude that all preferences consistent with the data must have x^0 preferred to x^1.

Given any x^0, it is possible to define the sets of x's that are revealed

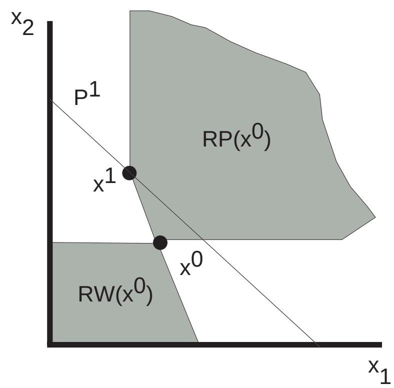

Figure 4: $RP(x^0)$ and $RW(x^0)$: simple case.

preferred to x^0 ($RP(x^0)$) and set of x's that are revealed worse than x^0 ($RW(x^0)$. A very simple example is shown in Figure 4. The possible set of supporting prices for x^0 must lie in the shaded cone so every such set of prices imply that x^0 is revealed preferred to the points in $RW(x^0)$. Similarly, the points in the convex hull of the bundles revealed preferred to x^0 must themselves be preferred to x^0 for any concave utility function that rationalizes the data.

Of course Figure 4 uses only one observation. As we get more observations on demand, we will get tighter bounds on $RP(x^0)$ and $RW(x^0)$, as shown in Figure 5.

Another approach, also suggested by Varian [1982a] is to try to compute bounds on *specific* utility functions. A very convenient choice in this case is what Samuelson [1974] calls the *money metric utility function.* First define the expenditure function

$$e(p, u) = \min \; pz \text{ such that } u(z) \geq u.$$

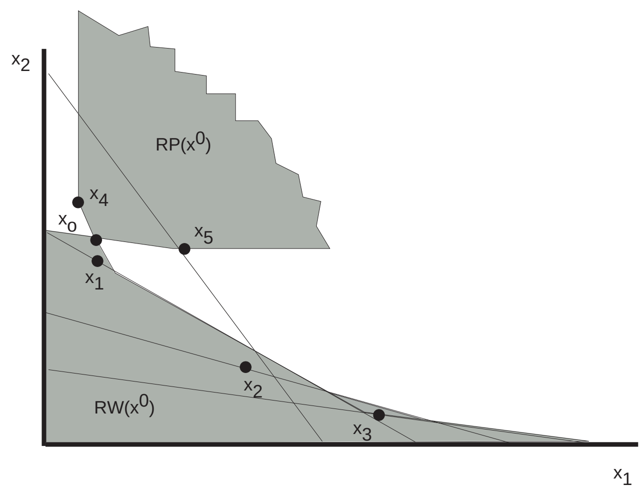

Figure 5: $RP(x^0)$ and $RW(x^0)$: a more complex case.

It is not hard to see that under minimal regularity conditions $e(p,u)$ will be a strictly increasing function of u. Now define

$$m(p,x) = e(p,u(x)).$$

For fixed p, $m(p,x)$ is a strictly increasing function of utility, so it is itself a utility function that represents the same preferences.

Varian [1982a] suggested that given a finite set of data (p^t, x^t) one could define an upper bound to the money metric utility by using

$$m^+(p,x) = \min \; pz^t \text{ such that } z^t R x.$$

Subsequently, Knoblauch [1992] showed that this bound was in fact tight: there were preferences that rationalized the observed choices that had $m^+(p,x)$ as their money-metric utility function. Varian [1982a] defined a lower bound to Samuelson's money metric utility function and showed that it was tight.

Of course, using restrictions on utility form such as HARP allow for tighter bounds. There are several papers on the implications of such restrictions in the theory and measurement of index numbers, including Afriat [1981], Afriat [1981], Diewert and Parkan [1985], Dowrick and Quiggin [1994], Dowrick and Quiggin [1997], and Manser and McDonald [1988].

9 Summary

Samuelson's 1938 theory of revealed preference has turned out to be amazingly rich. Note only does the Strong Axiom of Revealed Preference provide a necessary and sufficient condition for observed choices to be consistent with utility maximization, it also provides a very useful tool for empirical, nonparametric analysis of consumer choices.

Up until recently, the major applications of Samuelson's theory of revealed preference have been in economic theory. As we get larger and richer sets of data describing consumer behavior, nonparametric techniques using revealed preference analysis will become more feasible. We anticipate that in the future, revealed preference analysis will make a significant contribution to empirical economics as well.

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