

Analytical Models for Library Planning*

Operations Research models of the acquisition and storage functions of a library are developed. Rules for selection of materials for a depository are analyzed

and models of circulation interference and usage are explored. A generalized model of library costs and benefits is proposed.

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• Introduction

The process of managing libraries and networks of libraries is becoming more and more complex. This is partially due to the introduction of new technology and also due to an ever increasing quantity of published material to be processed and disseminated. In this paper, analytic tools are developed to help the decision maker resolve issues connected with storage and circulation policies. In the final section a generalized cost-benefit model of library operations is proposed.

In a previous paper (1) it was suggested that the work of Williams (2) and Raffel and Shishko (3) both have a similar underlying model. Assuming independence of items, a cost model for an information storage system has the form:

$$K(t) = k_1 + k_2t + k_3u(t) \quad (1)$$

In this equation, $K(t)$ is the total cost of holding one item for a period of t years; k_1 is the initial cost of acquiring the item; $k_2(t)$ is the holding cost which is linearly related to the retention period; and $k_3u(t)$ is the usage cost which is proportional to the number of uses made of the item during the period t .

This storage model for a single item does not take into

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account the effect of collection growth on the allocation of cost among the various items in a library. In practice, new items are arriving at a fairly constant rate (the so-called "law of libraries") and these items compete with older items for space and file maintenance. Under the assumption of declining use with age, the newer items will displace the older items because of the relative cheapness of their cost per use.

The following model is intended to demonstrate this effect by expanding the single item model to include exponential growth of a collection of items which have similar usage patterns. Since costs will be developed on a usage basis, those user or library costs which are directly related to the level of usage are omitted from the analysis to simplify the model. The use cost parameter, k_3 , could be included and would become a constant added to the total cost per use.

• Cost of Storing Many Items in a Growing Library

It has been observed by Dunn (4) and others that large university libraries have been growing in an exponential manner for a long time at a relatively constant rate. The data suggests a simple growth model of the form:

$$N_t = N_0e^{-at} \quad (2)$$

where N_t defines the size of the collection at a time t years in the past relative to the present size, N_o . The parameter " a " is a constant instantaneous growth rate, i.e., the ratio $-N_t'/N_t$. If the symbol α denotes the annual acquisition rate, then it is related to the instantaneous rate by the relation

$$\alpha = (N_{t-1} - N_t)/N_t = e^{\alpha} - 1 \quad (3)$$

or

$$\alpha = \ln(1 + \alpha) \quad (4)$$

It also can be shown that

$$\alpha = (\ln 2)/t_d = 0.693/t_d \quad (5)$$

where t_d denotes the doubling period or constant time it takes for the collection to double in size.

Dunn's study of the costs of operating large university libraries suggests a simple cost model of the form:

$$K_t = k_1 A_t + k_2 N_t \quad (6)$$

where K_t denotes the annual cost rate at a time t years ago. This is defined in terms of the annual number of acquisitions, A_t , at that time, multiplied by a unit cost k_1 , and the size of the collection at that time multiplied by a unit cost k_2 per volume per year. Since the annual number of acquisitions per year is related to the size of the collection by the annual acquisition rate, α , it is possible to rewrite equation (6) in the following way:

$$K_t = (k_1 \alpha + k_2) N_t = (k_1 \alpha + k_2) N_o e^{-\alpha t} \quad (7)$$

or in terms of the annual number of acquisitions, as:

$$K_t = [k_1 + k_2(1 + \alpha)/\alpha] A_t = [k_1 + k_2(1 + \alpha)/\alpha] A_o e^{-\alpha t} \quad (8)$$

These equations assume that the unit cost coefficients, k_1 and k_2 , remain constant over time, which is not very likely over any long period of time. A relatively simple and effective way to compensate for the change in unit costs over time, is to assume that these costs have been increasing at the same constant rate over time. If the instantaneous rate of change in the value of k_1 and k_2 is some constant k , then equations (7) and (8) should be modified as follows:

$$K_t = (k_1 \alpha + k_2) N_o e^{-(\alpha+k)t} \quad (9)$$

and

$$K_t = [k_1 + k_2(1 + \alpha)/\alpha] A_o e^{-(\alpha+k)t} \quad (10)$$

or simply

$$K_t = K_o e^{-(\alpha+k)t} \quad (11)$$

where K_o is the current annual cost rate. Thus, library costs are increasing faster than library size and both increase exponentially.

These models do not pretend to take into account any major innovations in methods and facilities for making inputs or maintaining collections. Such changes would be likely to cause jumps in the cost patterns with the exponential pattern resuming thereafter. Also, changes in the acquisition rate would cause exponential shifts in the growth patterns. However, the larger libraries have been

growing steadily for a very long time, and there has been relatively little change in the basic technology for a long time.

• Exponential Obsolescence and Circulation

As in the case of a single book, it has been observed that the usage rate of collections of books declines steadily from the time of their acquisition or publication. It has also been observed that the total usage or circulation of a collection tends to be proportional to the size of the collection over periods of time (5).

Assume that all items in the collection follow the simple exponential obsolescence pattern over time t

$$u'(t) = r e^{-bt} \quad (12)$$

$$u(t) = (r/b)(1 - e^{-bt}) \quad (13)$$

where $u(t)$ is cumulative use, r is a scale parameter associated with the instantaneous initial usage level and b denotes the instantaneous obsolescence rate (5). Also assume that the collection grows in an exponential manner. Then a fairly simple model of library circulation can be developed which agrees with both of the above observations.

If the instantaneous input to a collection at time t years ago is weighted with its usage rate at the present time, a measure of the contribution of that input to current circulation is obtained. If these contributions are summed or integrated over the last t years of growth, a measure is obtained of that part of the total circulation rate which is due to items acquired over the past t years. Let v_t denote the current circulation rate due to items which are less than t years old, then with reference to equations (12) and (2), this is defined as follows:

$$v_t = \int_0^t N_t' u_t' dt = \int_0^t r N_o e^{-(\alpha+b)t} dt$$

$$v_t = r N_o [a/(a+b)] [1 - e^{-(\alpha+b)t}] \quad (14)$$

where r is a scale parameter or the instantaneous initial usage rate of a new item. Its numerical value can be related to the average annual usage rate of the item during its first year since publication or acquisition.

As the period of collection growth over which v_t is computed becomes large, the total current circulation rate due to acquisitions during the period t approach a limiting value, which is defined by V_o where equation (15) states

$$V_o = r N_o a / (a + b) \quad (15)$$

that the total current circulation rate is directly proportional to the current size of the collection, N_o ; and the proportionality constant is a simple function of the rates of growth, obsolescence, and initial use. Figure 1 shows this pattern in the growth and circulation of the Purdue University Libraries as observed by Leimkuhler (6).

The ratio of equations (9) to (15) defines a cost per unit of circulation which can be denoted by $\bar{K}(V_o)$ and derived as follows:

$$\bar{K}(V_o) = K_o / V_o = (a + b)(k_1 \alpha + k_2) / r a \quad (16)$$

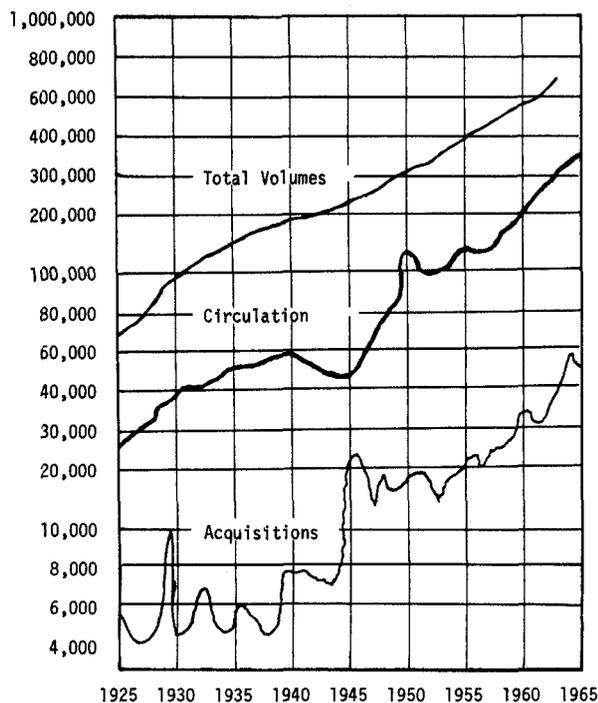


FIG. 1. Purdue Libraries.

Since the cost was assumed in equation (9) to increase exponentially over time because of the growth of the collection and because of external increases in the cost of inputs, the average cost per circulation will also increase because of the latter factor, i.e.,

$$\bar{K}(V_t) = \bar{K}(V_0)e^{-kt} \quad (17)$$

Here $\bar{K}(V_t)$ denotes the average cost per circulation at a time t years in the past.

• Library Cost When all Items Are Held for a Limited Time

In the previous sections it was assumed that all items were held indefinitely in the library. Many libraries find it desirable to discard inactive material or to transfer such material to a depository. Items to be discarded are selected on a basis of judgment, circulation histories, and age; or some combination of these factors. The age rule is perhaps the easiest to apply and is used commonly in the storage of periodicals and other serials where age is a major factor in identifying the item and in determining relative shelf location. The selection and discarding of little used items can be expected to add a certain amount of extra cost to the operation of a library. However, if it is done in a routine manner, these costs are likely to be partly proportional to the input and partly proportional to the storage cost. That is, they would be reflected in increases in the unit costs, k_1 and k_2 , which are used in equation (6). For this reason, an additional cost term can be avoided in the cost model for a library with limited retention time.

If all items are held for a period of t years and then sent elsewhere, the cost of operating the library could be expressed in the same form as equation (6), i.e.,

$$K(n_t) = k_1A + k_2n_t \quad (18)$$

where n_t denotes the number of volumes acquired which are less than t years old and A is the annual number of acquisitions. The variable n_t is defined by

$$n_t = N_0 - N_t = N_0(1 - e^{-at}) \quad (19)$$

Here, N_0 and N_t are defined in the same way as in equation (2). The circulation which is generated by these items which are less than t years old is defined by equation (14) as the variable v_t . By referring to equation (15), this can be written as

$$v_t = V_0[1 - e^{-(a+b)t}] \quad (20)$$

By solving equations (19) and (20) simultaneously so as to eliminate the argument t , the circulation of the restricted collection can be defined directly in terms of the number of items held, i.e.,

$$v(n) = V_0[1 - (1 - n/N_0)^{1+b/a}] \quad (21)$$

The inverse relation, which is used below, is

$$n(v) = N_0[1 - (1 - v/V_0)^{a/(a+b)}] \quad (22)$$

The subscript t has been dropped from the variables n and v for simplicity, since they imply a value of t according to equations (19) and (20).

• Minimization of Average Cost Per Use

According to equation (18) the library cost increases linearly with the size of the active collection, and according to equation (21) the circulation increases at a decreasing rate with the size of the active collection. This pattern is sketched in Figures 2(a) and 2(b), and is a typical kind of benefit cost relationship for a productive activity. In determining an optimal or near-optimal point for the planned operating level of the activity, it is well to consider the two possibilities v_1 and v_2 identified in Figure 2(b). The point v_1 represents the point of minimum average cost per unit of circulation or the maximum

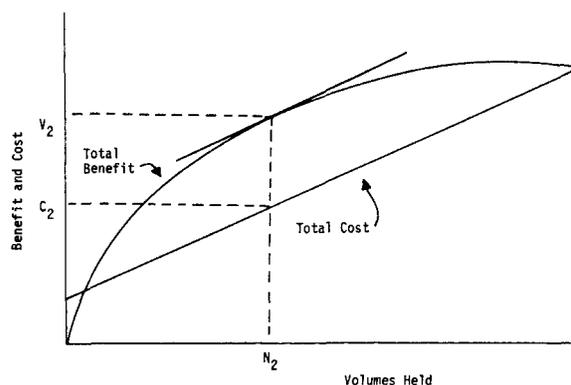


FIG. 2(a). Benefit and Cost vs. Holdings.

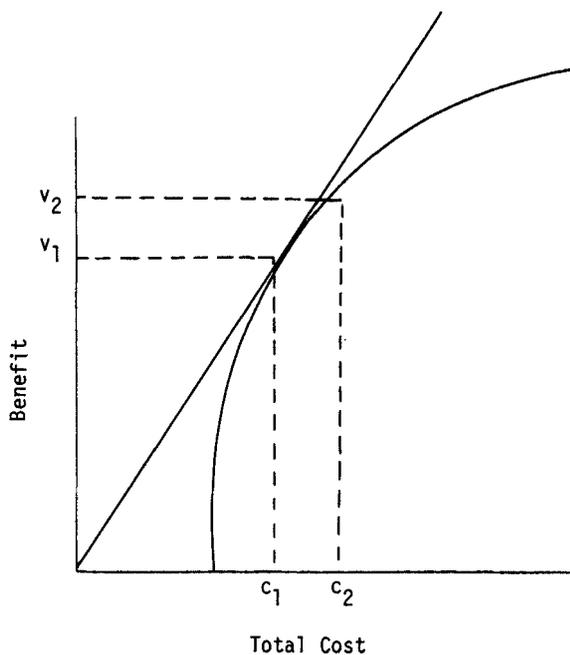


FIG. 2(b). Benefit vs. Cost.

average number of circulations per dollar spent. The point v_2 locates a higher level of activity where the marginal gain in circulation is less than the marginal cost. If the variable v were measured in dollars of net benefit, this would be a true optimum point, but since this is not the case there is no way of knowing the wisdom of choosing v_2 as the operating point. With v_1 there is the advantage of minimizing the possible diseconomies of the activity by establishing a technically efficient level of operation which can be controlled.

Upon substituting equation (22) into equation (18) library cost can be written as a function of circulation, i.e.,

$$K(v) = k_1A + k_2N_o[1 - (1 - v/V_o)^{a/(a+b)}] \quad (23)$$

Since the average cost, $K(v)/v$, per unit of circulation achieves a minimum value at the point where it equals the marginal cost, this point can be found by setting the derivative of equation (23) equal to the average cost and solving for the level of circulation, v_o , which is found to be:

$$v_o = V_o(1 - [k_2/(k_1a + k_2)]^{1+a/b}) \quad (24)$$

By substituting this value for v_o into equation (20), the holding time, t_o , which minimizes the average cost per unit of circulation is:

$$t_o = (1/b)2n(1 + k_1a + k_2) \quad (25)$$

Also, the size of the active collection, n_o , which minimizes average cost can be obtained by substituting equation (24) into equation (22). This yields the relationship

$$n_o = N_o(1 - [k_2/(k_1a + k_2)]^{a/b}) \quad (26)$$

The ratio n_o/N_o is the fraction of total past acquisitions which are retained in the active collection, and the ratio

$(1 - n_o/N_o)$ measures the percentage reduction in annual storage costs due to the disposal of inactive items.

As an example of the implications of this model, consider a collection which is growing at a rate of about 5 percent per annum and is subject to an obsolescence rate of approximately 0.05. Also, assume that the cost of a new acquisition is about 20 times the cost of holding an item one year. Under these conditions, items should be held in the active file for about 13 years, which accounts for about half of the total past acquisitions and about three-fourths of the total potential circulation from past acquisitions. This would imply also that at an optimal level of operations, two-thirds of the total cost would be devoted to the acquisition or input effort and only one-third to the retention or storage effort.

• Retention of Inactive Items and Age Rule Depositories

The above model focuses on holding time as the control variable in storage, and emphasizes the economic advantage of discarding inactive materials. Because of this, it appears to ignore some important practical questions about providing access to relatively inactive items. An important assumption in the model is that all items in the collection can be represented by a single exponential obsolescence pattern. If different items have different obsolescence rates they should be retained for different periods in order to minimize their average cost per use. According to equation (25) the optimal holding time is directly proportional to the inverse of the obsolescence rate, i.e., if the rate is doubled then the holding time should be halved and vice versa. In practice, it can be observed that technical libraries which are said to exhibit the highest obsolescence in their usage are also more likely to pursue a vigorous discarding policy and are more likely to think in terms of a limited size for the total collection. The opposite is supposedly true for libraries in the humanities.

An obvious way to increase the optimal retention time for an item is to reduce the unit cost parameter, k_2 , which includes all library costs other than those connected with the input of new items. Equation (25) indicates that the effect of reducing k_2 is not nearly as direct as the effect of decreasing the obsolescence rate. In practice, libraries have attempted to reduce holding cost in two principal ways: by increasing the size of collections and achieving certain economies of scale, and by use of depository or overflow libraries where inactive items can be shelved more compactly and in less expensive space. These methods tend to increase the cost of access for both the user and the library. From the standpoint of the storage model, it is important that the reduction in space cost is not cancelled out by the increase in retrieval cost. Raffel and Shishko (3) seem to indicate that this may be the case with the use of depository-type storage at M.I.T.

There appears too little evidence that the use of this kind of facility can achieve dramatic reductions in cost, although there is need for more hard data to support this conclusion.

The same problem of increasing retrieval costs while reducing space costs appears to be delaying the acceptance of miniaturization as a long run solution to the problem of providing access to inactive materials. A more successful working solution is reported in England in the establishment of a National Lending Library service to back up local libraries. This service is provided on a formal pricing basis with established costs of time for the user and of money for the borrowing library. The formal pricing structure helps to both insure the recovery of cost and to control the demand so that a more stable utilization of materials results. The report by Williams argues for the establishment of similar lending services in the U.S.A. on a regional or national basis (2).

• Age Rule Depositories

Where space is at a premium, depositories offer a rational approach for suboptimizing the problem of storage. Two commonly used rules for selecting items for storage are the age rule and the current usage rule. The latter rule can be based on the time since last use or the average usage over a specified period. These rules may be used in various combinations, and may be modified according to certain characteristics of the material, such as language, and the opinion of users or librarians. The age rule is the simplest rule to apply and is often used to select journals for storage.

Under the assumption of exponential obsolescence and growth, equation (21) provides a simple relationship between the size and the activity of an age rule depository. If the symbol m is used to denote the proportion of the collection stored in the depository, and w denotes the proportion of total usage from the depository, then

$$w = m^{1+b/a} \quad (27)$$

If the obsolescence rate b is equal to the accession rate a , then w is the square of m ; and if b is greater than a , w is a higher power of m , that is, w is considerably smaller since we are dealing in fractions. As the library grows in total size, if the retirement age is kept constant, the fractions w and m remain constant although both are growing exponentially in absolute terms.

The above model can be used to show the effect of restricting the size of an active collection to a predetermined level. When this level is reached, the depository would begin to grow exponentially, and the fraction of the collection stored after t years would be defined by:

$$m_t = 1 - e^{-at} \quad (28)$$

The fraction m_t grows at a decreasing rate until virtually all of the collection is stored in the depository, assuming there is no change in the policy. If the items exhibit

simple exponential obsolescence and are selected on a basis of age, then the retirement age, d_t , would have to diminish as time goes on and would be defined as follows:

$$d_t = -(1/a) \ln m_t = -(1/a) \ln(1 - e^{-at}) \quad (29)$$

The fraction of total library usage generated by the depository would increase exponentially according to the relationship

$$w_t = m_t^{1+b/a} = (1 - e^{-at})^{1+b/a} \quad (30)$$

If the parameters a and b are equal, equation (30) reduces to the simpler relationship

$$w_t = m_t^2 = 1 - 2e^{-at} + e^{-2at} \quad (31)$$

This equation is evaluated in Table 1 for various values of t .

Table 1 demonstrates the effect of limiting the size of active collections. It is interesting to observe the delayed effect on the usage of the depository as compared with rapid initial growth of the percentage of volumes in the depository. The size of the depository is only dependent on the growth rate of the collection, while the use of the depository is dependent on both the growth rate and the obsolescence rate. The values in Table 1 hold for the case where an age rule is used and the rates are equal to 0.05; but the pattern would be much the same for other rates and selection rules. In the long run the depository would eclipse the active collection. It is of interest to note here a conviction, held in special library circles, that technical libraries tend to maintain a rather stable size over time, some say at about 20,000 volumes. This is done by discarding inactive items and would imply that the discards would eventually exceed the active collection by a considerable amount. Since these discarded items are likely to be needed at some future time, the fixed size technical library would need to be backed up by a large depository library. Industrial libraries tend to use university libraries for this purpose, and this is one of the roles of the National Lending Library in Science and Technology in England.

It is unlikely that the simple exponential model provides a sufficient explanation of the actual patterns of

TABLE 1. Depository Size and Usage when the Active Collection is Held at a Fixed Level and the Rates a and b Equal 0.05

Year after Start of Depository, t	Rel. Size of Depository, m_t	Rel. Usage of Depository, w_t	Retirement Age, d_t
1	0.05	0.003	60 yrs.
5	0.12	0.01	42
10	0.39	0.15	19
15	0.53	0.28	13
20	0.63	0.40	9
25	0.73	0.53	6
50	0.92	0.85	2

inactivity of library materials, or that the age rule would work well without some compensating modifications. Trueswell (7), Lister (8), and Morse (9) have shown that usage rules do a better job of selection than do age rules. Jain (5) has examined retirement policies based on the probability of item inactivity, and Gurk and Minker (10) developed a model to estimate the storage requirements for a data bank where retention is a function of age and usage. Such models are facilitated by computer-based circulation control systems.

• Acquisition Delays and Item Usage

The larger part of library operating costs goes into the initial cost of adding new items to the collection rather than retention. The larger part of input cost is the cost of selection and processing rather than the purchase price of materials. While the high cost of adding new materials is a major concern in library budgeting, of more importance to library users is the long time delay in making new items available for perusal. Morse (9) has given considerable attention to the urgency of making new items available during the first few years of publication when demand is at a peak level. A closely related problem is how to identify quickly those items which need duplication, since any long delay loses most of the advantage in duplication and magnifies the disadvantages. Quick accession provides an important opportunity for increasing the benefits to be gained from a library item, especially if it can be done without a substantial increase in the accession cost. Some university faculty members identify accession delay as the most important deficiency in current library service.

The significance of acquisition delays can be easily seen by considering the case where a library is growing exponentially and the input delay time is constant. Let h denote the input delay time for the ordering, receiving, and cataloging processes, i.e., the time from publication to first use, and let $I(h)$ represent the inprocess inventory of all such items. Then $I(h)$ can be defined by the relationship

$$I(h) = N(1 - e^{-ah}) \quad (32)$$

where N is the size of the inprocess and shelved collection and a is the growth rate. Since N is growing exponentially, $I(h)$ is also growing exponentially as long as h is held constant. This is shown graphically in Figure 3.

There is reason to suspect that equation (32) actually understates the size of the in process inventory since in order to maintain a constant input time it is necessary to expand the capacity of the operations at the same rate as the work load increases. Dunn (4) indicates that the expansion in library labor is at a somewhat slower rate than the growth in volumes acquired. This could indicate either that the work capability is not keeping pace with the load or that labor efficiency is being in-

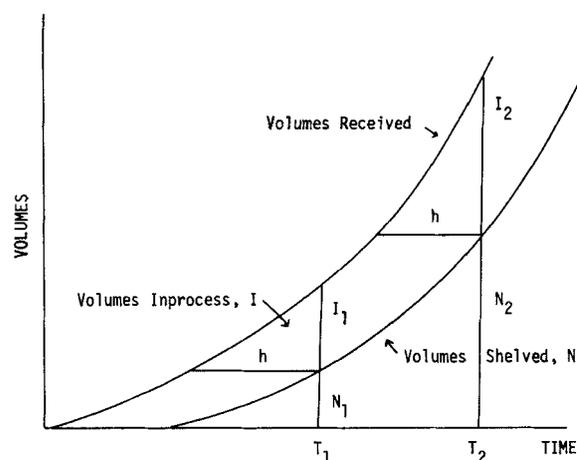


FIG. 3. Inprocess Inventory Due to a Fixed Accession Delay Time, h , when a Collection Grows Exponentially.

creased because of economies of size or the introduction of new methods. A second reason for expecting equation (32) to underestimate the size of the inprocess inventory is the presence of random and periodic variations in the work load. Under conditions of peak loading and ever-increasing scale, one would expect the variations in loading to cause considerable interference and congestion in processing which should add to the size of the in process stock.

Under the assumption of a common exponential obsolescence pattern for the collection, a fixed input delay interval would cause a "loss" in a certain amount of potential usage which would be a fixed proportion of the actual circulation. This proportion, q , can be defined as follows:

$$q = e^{-(a+b)h} - 1 \quad (33)$$

For relatively small values of the exponent, $(a+b)h$, the value of q is approximately equal to the value of the exponent. For example, if a and b both equal 0.05 and h equals 0.5 years, then q is approximately five percent of actual usage. This is likely to be a gross underestimate of the actual case, however, since Morse has shown that the obsolescence rate of new materials is considerably higher than 0.05 and can even be as high as 0.50 for some scientific items. This would indicate that the ratio q could be as high as 0.30 or almost a third of the actual usage, if h is about six months. Morse found that 80 percent of the total ten year usage of a book occurs in its first three years in the library. He urges the development of methods of quickly identifying and acquiring original and duplicate copies of high use items as a particularly important way to improve library effectiveness.

• Circulation Interference and Usage

An important factor contributing to user cost and to the level of usage of a collection is availability of items

which have been added to the collection but may be missing from the shelf for a variety of reasons. Various people have estimated that the probability of a requested item being on the shelf at the time of request varies from 0.50 to 0.67 in many libraries, although some librarians say this is as high as 0.75. This does not mean that only that proportion of the collection is on the shelf, but that that proportion of the requested items are available. Thus the measure is heavily weighted by the absence of the more popular items. Buckland and Hindle (11) have argued that the unavailability of popular and well used items not only inconveniences those patrons requesting them, but creates a shelf bias toward inactive and perhaps less relevant materials for browsing and for those patrons who must use the material available. The two basic methods of increasing the availability of a collection are duplication and tighter circulation control.

Duplication of popular items increases the availability of the items but actually decreases the circulation rate per volume of the library. While duplicates may be acquired at less cost than original items, because they have already been catalogued, it is unlikely that this reduction in cost is sufficient to prevent duplication from increasing the average cost per use for the collection. Leimkuhler (6) developed an elementary queuing-type model which shows the effect of duplication on circulation. In general, the model shows that two copies of an item can never succeed in doubling the circulation rate per volume. For example, if a single copy is used so that it is available only half the time, then duplication will only increase the total circulation for the two volumes 60 percent, which reduces the average circulation rate twenty percent for the two volumes. At the same time, the *availability* rate for the title would be increased from 0.5 to 0.8 by duplication according to the model. This model can also be used to show the effect of duplication among branch libraries, which in effect divides the demand between two locations. In general, it can be shown that duplication at two locations yields less circulation per volume than does duplication at a single location. It also fails to produce as great a level of availability at the two locations than at a single location. This model is particularly interesting because it demonstrates a case where there is a trade-off relationship between user and library cost, that is, the user cost in terms of availability is reduced by increasing the cost per use to the library.

Buckland and Hindle (11) in their study of availability concluded that improved circulation control may be a better approach than duplication in centralized libraries. Circulation control can range from no-loan policies, through various limited loan policies with recall and renewal options, to unlimited loans. They found that when loans are made for periods of a week or more, the return of the material and its probability of renewal was independent of the loan period. This suggests that usage can be confined to a relatively short period without penalizing most users. Buckland and Hindle used a

simulation model to show that a good policy for a library to follow is to restrict loan periods to one week for the most popular items in the collection, and to allow end-of-term loans for the balance of the collection. The latter policy has the advantage of simplifying the date control and notification procedures. The simulation results indicate that this policy should produce a relatively high level of general availability and a low collection bias when about ten percent of the collection is placed on a short loan status.

It is interesting to note that under the assumptions of exponential growth and obsolescence, the most popular ten percent of the collection would be those items acquired within approximately ten years, if the acquisition rate is about five per cent per annum. Actually, Buckland recommends monitoring the average number of recorded uses per year as a guide to the selection of short loan period items. These results seem to agree substantially with the finding of Morse (9) from his extensive study of the M. I. T. Library. Morse takes an analytic approach rather than a simulation approach by drawing heavily on the theory of stochastic process and queues. He suggests the use of a one week period for popular items, and also notes the importance of giving greater attention to the time it takes to reshelve these items and to replace missing items. In general, the evidence indicates that the popular items in a library, which are likely to be the newer items and a relatively small proportion of accumulated holdings, should be handled in a different manner than the rest of the collection. Such a division in the document control procedures can greatly increase the effectiveness of a library without increasing library cost excessively. The implementation of such a policy is much easier if a computer-based data processing system can be used to control document circulation and location. A more interesting question is whether the benefits to be gained from a more sophisticated document control system can be sufficient to underwrite the cost of introducing mechanized data processing.

• Generalized Models of Library Costs and Benefits

These models have been deliberately simplified to facilitate the analysis of certain operating policies and to help develop a sound approach to the study of library costs. Operating cost is defined as the sum of three component costs which relate to the acquisition, storage, and circulation functions of a library. A more general cost model should recognize all of the important options which are available in the exercise of these three functions. A useful approach to the identification of these options is to divide them between those options which are related to the scale of an activity and those which are related to how the activity is performed. This is the distinction in microeconomics between how a product is made and how much of the product to make. The first is a more tech-

nical question and depends on the ways in which inputs can be combined to produce a product, and the supply of the inputs. The second question is more of a marketing problem and depends on the answers to the first question and the demand for the product. In practice, of course, this division is not so neat, nor is a manager free or willing to explore all possible options.

In the above models, the principal scale factors were the levels of acquisitions, the volume of stored documents, and the amount of usage. When the assumptions of exponential growth and obsolescence are made, it is possible to tie these three together in a deterministic manner, so that the specification of one level determined the others. The methods for performing the three functions or the technology factors were reflected in the cost coefficients, k_1 , k_2 , and k_3 ; and also in such control variables as the holding time t_h , the accession delay time, t_a , and the user access time, t_u . Most of the analysis focused on the single control variable, holding time.

A generalized model of library costs could be written in the following manner:

$$C(S,T) = C_1(S,T_1) + C_2(S,T_2) + C_3(S,T_3) \quad (34)$$

where S designates scale variables and T designates technology variables, and the subscripts 1, 2, and 3 designate acquisition, storage, and usage functions. The cost, $C(S,T)$, of operating a library with a particular technology, T , at a scale, S , is the sum of the three component costs, C_1 , C_2 , and C_3 . The complete definition of the technology of a library in a manner which reveals all of the possible factors which might be varied so as to reduce costs, is a very difficult task. Normally, one focuses on a few variables at a time and assumes that all other factors are fixed and can be represented by appropriate parameters and functional relationships. It appears that the technology factors of interest are those which have either a space or time dimension, such as acquisition, storage, and retrieval time, or shelf space, reading space, and user proximity. There are many others which are of interest and even these might be subdivided into smaller time or space components.

Scale factors and measures can also become quite involved and complicated, when one wants to define and manipulate them in an analytic manner. Scale is a relative measure and the base chosen is important. For example, the acquisition effort might be described by the actual number of items acquired or in terms of the potential number which *might* have been acquired, or in terms of the actual number screened. Similarly, usage might be measured in terms of actual uses, actual requests, or potential requests. Collection size could refer to those on hand, those on hand and in use, and those held relative to what *might* be held. Besides these distinctions, there is the difficult problem of aggregation and the avoidance of adding dissimilar things. From some viewpoints, everything in a library is a "one of a kind" item, every use made is a unique entity, and every acquisition is a special problem. Such distinctions greatly en-

large the problem of analytic representation and force the use of computers and computer methods to handle the sheer bulk of the analytic detail required.

The problems of cost measurement and cost modeling in libraries, for all of their difficulty, are not nearly as formidable as the problems of benefit measurement and modeling. The same distinction made between scale factors and technical factors in the modeling of costs seems to apply to the measurement of benefits. Library benefit can be thought of as a difference measure between user reward and user cost to obtain the reward. User rewards relate to user needs which are outside the direct control of a library, although the library may have an influence in determining which needs the users seek to satisfy. Library benefit is more directly related to the library's ability to reduce the cost to users of need satisfaction. This is done by increasing the probability of satisfaction and reducing the time and effort required. The latter element of user cost, time and effort, is related closely to the technological design of the library. The level of satisfaction is the scale component in user cost, and is related to the scale of the library. Thus, it is argued that the larger the collection in a field, the greater the likelihood of its satisfying requests, although it might require more time and effort from the user. Furthermore, there are likely to be diminishing returns with increased size due to obsolescence and increased "scattering" of references in peripheral locations. How to strike meaningful balances between availability, collection size, and access time is one of the most difficult questions in library planning.

• Summary

The models presented in this paper explore the phenomena of exponential growth and obsolescence of a collection of library materials and their effect on acquisition, storage and circulation.

Among the issues discussed are the cost of storing items in a library, the effect of exponential obsolescence on circulation, rules for selection of materials for storage and equations characterizing acquisition delay, and circulation interference.

It is shown that when all items are transferred to a depository after a fixed number of years, there is a holding time which minimizes the average cost per circulation. In addition relationships between library and depository size and usage are derived, as well as relations between acquisition delay time and the size of the in process inventory. By handling more popular library materials in a restricted manner, such as allowing a shorter circulation period, the models suggest circulation interference will be alleviated.

Finally a formalism for a general cost-benefit analysis has been suggested in the last section. How to develop an analytic solution to this problem remains both a difficult and important task for library systems analysis.

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