Antitrust Scrutiny of Price-Fixing Clauses in Patent Licenses

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Patent licenses are frequently subjected to antitrust scrutiny, especially when they contain restrictions beyond simple fixed fees and royalties. Clauses that fix the price at which a product is sold have proven contentious, upheld by the courts in some circumstances and rejected in others. No agreement has yet emerged on how to reconcile court decisions and economic rationale. We study the effects of price-fixing with a stylized model featuring heterogeneous consumers choosing between two product generations, sold by firms that compete on price. Licenses can be compared according to how much reward is collected by the inventors. In our example license schemes, we find that royalties and fixed fees are not enough to make a license profit neutral, even with zero-cost manufacturing. The erosion of profit when price-fixing is forbidden depends on the quality gap between generations of product and on whether the license is based on flat royalties or a percentage rate. A percentage-rate license has particularly poor performance for small quality gaps, as total profits approach zero in the limiting case. A flat-royalty license is nearly optimal in this case, but may be more difficult to negotiate if there are information asymmetries. For large quality steps, a percentage rate license is a better choice, with nearly no erosion of profit. In all cases, price fixing can be used to restore optimal profits. In general, only the price of one product must be fixed, since the other can be controlled with royalties.

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1. Introduction

On the surface, a tension exists between the bodies of patent and antitrust law. While patents give limited monopolies to new inventors, if it were not for the patent right, those same monopolies would violate the antitrust laws. The effect of this tension is that patent holders are closely scrutinized to see if they overstep the boundary of power intended by the patent act. In the words of Baxter, "what is not authorized is forbidden." [2]

The boundary between patent and antitrust is difficult to isolate from often cryptic court opinions and appears to shift over time, complicating analysis. Several attempts have been made to find simple rules that govern the relationship between the two institutions. Gifford argues that a paradigm is emerging in which patent law trumps antitrust [4]. On the other hand, Kaplow and others conclude that current law is more complicated, or even uncertain than this allows [6]. Most scholars agree that patent and antitrust concerns must somehow be balanced.

Attempts at balancing sometimes focus on developing a set of per se rules that can be mechanically applied. This is exemplified by the nine no-no's promulgated by the Department of Justice in the 1970's, a list of licensing practices that once triggered automatic antitrust scrutiny [14]. However, the 1995 Antitrust Guidelines for the Licensing of Intellectual Property replaced the nine no-no's with a more general set of criteria [11], recognizing that situations arose in which each practice might be pro-competitive. An alternative to per se rules is the so-called rule of reason, though critics claim it is often an excuse for arbitrary decisions [1].

Most recently, Scotchmer and Maurer have proposed a set of criteria for judging licensing behavior that largely seems to fit with court decisions [7]. Restating their findings, a valid license must conform to

1. Profit Neutrality – the reward available to an inventor should not depend on whether she has the necessary manufacturing capacity to make the product herself, or if she licenses another party to manufacture the good.
2. Derived Reward – the inventor should receive compensation derived directly from the value created by her invention.
3. Minimalism – a license should not include extra restrictions beyond those required to achieve profit neutrality and derived reward. [7]

Though grounded in economic incentives, these guidelines are relatively easy to apply because they don't require a comparison of ex ante incentives to ex post deadweight loss. Scotchmer and Maurer use these principles to evaluate the legality of price fixing clauses in patent licensing. In a model they study, a patent holder and a licensee both have manufacturing capability, and both face increasing marginal costs of production. After a license agreement is fixed, the two firms participate in a game of Cournot competition. Under such conditions, licenses employing only royalties and fixed fees are not sufficient to distribute manufacturing efficiently. On the other hand, by contractually fixing the price at which the good is sold, the patentee can enjoy the optimal profit she would get if she owned all manufacturing facilities. The lesson is that if costs of production are sufficiently non-uniform, price fixing should be permitted, since fixed fees and royalties alone will not lead to profit neutrality [7].

In this study, we extend the model of Scotchmer and Maurer by considering a heterogeneous population of consumers with varying sensitivity to product quality. We also replace their assumption of Cournot competition with price competition. Our extension is sufficiently rich to demonstrate some novel results:

First, we show that even if manufacturing is zero-cost, competition between product generations generally distorts profits as compared to the case in which inventions are held by one firm. We find that a percentage-rate license and a flat-rate license exhibit different characteristics: When the quality gap is small, a flat-rate license is nearly profit neutral, but total profits are reduced to zero for a percentage rate license in the limiting case. For large quality gaps, a percentage rate license is close to optimal, but a flat rate license is impractical, since most value must be transferred through fixed fees. For intermediate product gaps, both types of licenses are substantially not profit neutral.

When equilibrium profits are sub-optimal, price-fixing is one additional instrument that can remedy the situation. Perhaps surprisingly, we find that fixing either the price of the upstream good or the price of the downstream good is sufficient – the other price can be controlled with royalties. Along the way, we also demonstrate a way to achieve profit neutrality using cross royalties, but suggest that this probably violates the derived reward principle.

The rest of this paper is organized as follows: The next section is a review of two fundamental Supreme Court decisions regarding price fixing. Section 3 presents the basic components of our model, highlighting the novel aspects we contribute. Section 4 presents three sample licenses, characterizes their behavior, and evaluates instruments that can be added to make these licenses profit neutral. Section 5 discusses applicability of the model to court decisions. Section 6 is a brief conclusion.

2. Two Relevant Court Decisions

The Supreme Court has considered the legality of various license terms vis-à-vis antitrust law on several occasions. In *U.S. versus General Electric Company*, the Court upheld a price fixing clause in a patent license [12]. In the 1920's, General Electric held key patents in the field of tungsten filament light bulbs. They granted a license to Westinghouse Electric and Manufacturing Company, under which Westinghouse was bound to price their light bulbs no lower than General Electric was pricing the bulbs it produced internally. The Court upheld the license, but did not mention manufacturing costs in its argument, rather focusing on General Electric's profit:

> It would seem entirely reasonable that he should say to the licensee, 'Yes, you may make and sell articles under my patent but not so as to destroy the profit that I wish to obtain by making them and selling them myself.' [12]

Since General Electric presumably doesn't care whether profits come from direct sales or royalties, Scotchmer and Maurer conclude that the court's reasoning is economically inadequate, but its conclusion nevertheless embodies the profit neutrality principle [7].

At other times, the Supreme Court has invalidated price fixing clauses. In *U.S. v. Line Material Company*, the Court considered a more complex licensing system involving several actors [13]. In the 1930's, Southern States Equipment Corporation developed and patented a circuit breaker with a replaceable fuse. The circuit breaker's operation was complex and involved a solenoid-powered release mechanism. A competitor, Line Material Company, improved on Southern's design, replacing the solenoid with a simpler gravity-powered mechanism. Line Material received an improvement patent for its contribution, resulting in a blocking situation.

To resolve the block, Line granted Southern a non-exclusive license to make and sell the improved gravity-powered fuse. In return, Southern granted Line a license to exercise the upstream patent, and an exclusive license to sublicense the patent. Line negotiated separate agreements with several third-party manufacturers to produce the gravity-powered fuse, and the royalties from these agreements were to be evenly split between Southern and Line. Finally, each agreement gave Line Material the power to select a single price at which each manufacturer would sell the fuses.

The Supreme Court decided not to permit price-fixing in this context:

> Where two or more patentees with competitive, non-infringing patents combine them and fix prices on all devices produced under any of the patents, competition is impeded to a greater degree than where a single patentee fixes prices for his licensees. The struggle for profit is less acute. Even when, as here, the devices are not commercially competitive because the subservient patent cannot be practiced without consent of the dominant, the statement holds good. [13]

Scotchmer and Maurer again argue that the Court's economic reasoning is flawed, but suggest that the decision might be correct if marginal costs of production are approximately constant and equal for each firm [7]. In such a scenario, any distribution of manufacturing is efficient.

3. The Basic Model

We expand on a classic example, explored by Green and Scotchmer [5], in which there are two generations of invention -- for example, Southern's solenoid-driven fuse followed by Line Material's gravity-driven improvement. Green and Scotchmer notice, and Denicolò and Zanchettin confirm [3], that patenting and licensing opportunities determine the expected division of profits between the two inventors. Hence, the ex ante incentive to innovate depends upon the range of possible agreements into which firms can enter.

A limitation of these models is that the total profit flow is exogenously fixed, not dependent on the firms' pricing decisions. This means that most licenses artificially appear profit neutral. To overcome this difficulty, we introduce a heterogeneous set of consumers that have to decide between the generations of product. We first describe the consumer model, before returning to the firms.

Consumer Model

Closely following the assumptions of O'Donoghue, et al. [8], we imagine a continuum of consumers who face a choice between different varieties of a product in a single-stage game. To each product variety, we assign a quality, $q \geq 0$, and we assume each consumer either purchases one product or nothing at all.[1] We normalize the mass of consumers to 1, and index each consumer by type, $\theta \geq 0$, such that a type θ consumer gains linear utility, $\theta q - p$, for choosing a good of quality q at price p, and utility 0 if she purchases nothing. We assume that consumer types are distributed according to a continuous density function, $f(\theta)$, with cumulative distribution F. Finally, we need a technical assumption, that f has increasing hazard rate (IHR),

$$\text{IHR Assumption:} \quad h(x) = \frac{f(x)}{1 - F(x)} \text{ is increasing.} \tag{1}$$

This assumption holds for normal, exponential, and uniform distributions, for example.

Alternately, the IHR assumption can be expressed by saying that the function 1-F has decreasing elasticity. In our model, 1-F serves as a generalization of a demand function, and in fact reduces to a regular demand function in the case of a single product. To demonstrate this, consider a single product of quality q, sold at price p. A consumer of type θ derives utility v_θ, given by,

$$v_\theta = \begin{cases} 0, \; \textit{if purchases no product} \\ \theta q - p, \; \textit{if buys product} \end{cases} \tag{2}$$

The consumer's optimal strategy depends on whether $\theta q - p$ is positive or negative and is given by,

$$\begin{aligned} &\textit{buy nothing, if} \quad \theta \leq \frac{p}{q} \\ &\textit{buy product, if} \quad \theta > \frac{p}{q} \end{aligned} \tag{3}$$

We will find it convenient to depict the consumer's choice as a function of θ on a graph, as below, where M stands for a monopoly configuration.

[1] This begins to distinguish our model from that of O'Donoghue, et al., who assume all consumers purchase one good.

Configuration M.

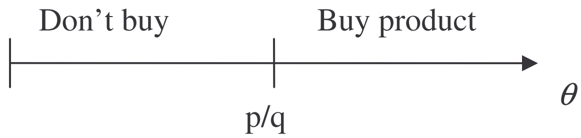

Figure 1

If a single firm supplies the product, its revenue can be found by integrating to get the number of consumers who decide to buy:

$$\pi = p\int_{p/q}^{\infty} f(\theta)d\theta = p\left[1 - F\left(\frac{p}{q}\right)\right] \tag{4}$$

So it is clear that 1-F acts like a familiar demand function, horizontally scaled by quality, q. The IHR assumption can be used to demonstrate that there is a unique maximizing price (scaled by quality), p_m/q, which we call the monopoly price, and we will refer to the corresponding profit as the monopoly profit for a good of quality q.

Firm Model

Ultimately, we are interested in the ex ante incentives to produce innovation. We can imagine a multi-stage game, in which two firms arrive in sequence and decide whether to invest in research at some cost, aware of the expected profit flows that will result from licensing. Indeed, we keep such a game in the backs of our minds as we proceed, but we will suppress the first stages, which are well-explored in [5], and focus only on the subgame in which both firms have already created their inventions and are prepared to enter a license agreement.

With this in mind, we imagine two firms: Firm 1 holds a patent for the upstream product 1, which has quality q_1; firm 2 knows how to create an improved product 2, of quality $q_2 > q_1$. In order to protect its improvement, firm 2 may take out a patent on it, resulting in a blocking patent situation. Alternately, we may assume that firm 2 simply holds its improvement as a trade secret. We believe that our model applies well to both cases, but there are some important differences, outside the scope of the mathematical analysis, that should be noted.

When firm 2 holds a trade secret, an information asymmetry results. There is a danger that firm 2 will reveal its secret to firm 1 in the course of negotiating for a license, and firm 1 will simply take the secret and start manufacturing improved products without compensating firm 2. Although the legal system includes some safeguards against this, it may prove practically impossible for firm 1 to avoid using the knowledge it received from firm 2 during negotiation.

If missing crucial information about firm 2's operation, firm 1 will not be able to accurately predict the demand for product 2 and firm 2's profit margins, so it will be difficult to agree on license terms. In such a situation, firm 1 may prefer a contract in which firm 2 pays some fixed fraction of its sales revenues in royalties. We call this a percentage rate license, and we will characterize its behavior, below.

Finally, if firm 2 holds a trade secret, it is not protected in the event that another firm independently generates the same idea. This threat may constrain the price that firm 2 charges. We consider each of these effects to be second-order, and we believe that our model is sufficiently general for both the case where firm 2 holds a patent and the case where firm 2 retains a trade secret.

However firm 2 is protected, its improvement is blocked by firm 1's patent, so a license is needed in order for firm 2 to produce product 2. After firm 2 gains permission to produce, we will denote the price at which firm 1 sells product 1 by p and the price at which firm 2 sells product 2 by r. Finally, we assume zero marginal costs of production. This simplifies analysis, and allows us to highlight the effects of price competition. The effects of production costs are already well studied in [7].

4. Behavior of Three Licenses

In this section, we argue for our central proposals:

1. *With two generations of product and heterogeneous consumers, a license based on fixed fees and royalties is not profit neutral.*
2. *Different licenses exhibit different performance characteristics. In general, the amount of profit erosion depends on the type of license and on the quality step between product generations.*
3. *Price fixing is one license element that will provide profit neutrality. A license that fulfills the principle of minimalism will only fix one price, since the second can generally be controlled with royalties.*

We evidence these proposals by considering three different licensing strategies. Before we can decide whether each license is profit neutral, we must have a reference point to which we can compare profits. Since firms can redistribute profits between themselves using fixed fees, only the total profit for both firms matters in making this comparison.

4.1 Base Case: Inventions Consolidated in One Firm

It is not immediately clear what the benchmark should be for determining profit neutrality. We agree with Scotchmer and Maurer that the best choice is the profit that would result if both inventions were held by a single firm [7]. This is the maximum total profit that can be generated.

There is an intuitive argument for this benchmark: From the perspective of firm 2, not owning the upstream patent is in a sense an "accident of history." The incentive for firm 2 to create the improvement is already much lower than it is for firm 1, since firm 2 must split the extra available profit with the other firm. Lowering the total profit would further unbalance the incentive for firm 2 to invest. Because of this, we believe that the maximum total profit holds most closely to the spirit of profit neutrality.

Another potential benchmark that might seem attractive is the profit that would result from free competition between the two products. This is accomplished, for example, in the lump-sum license explored below. Unfortunately, as we discover in the next section, free competition can drive total profits below the profit firm 1 would get by simply blocking firm 2 from the market entirely. This has the pathological effect of removing all incentive to invest in certain improvements.[2,3]

To characterize our benchmark, suppose that a single firm sets prices *p* and *r* for product 1 and product 2, respectively. Then a consumer of type θ experiences utility given by,

$$v_\theta = \begin{cases} 0, \; if \; purchases \; no \; product \\ \theta q_1 - p, \; if \; buys \; product \; 1 \\ \theta q_2 - r, \; if \; buys \; product \; 2 \end{cases} \tag{5}$$

Maximizing her utility, the consumer's strategy will be,

$$\begin{aligned} &buy \; nothing, \; if && \theta \le \frac{p}{q_1}, && \theta \le \frac{r}{q_2} \\ &buy \; product \; 1, \; if && \theta > \frac{p}{q_1}, && \theta \le \frac{r-p}{q_2-q_1} \\ &buy \; product \; 2, \; if && \theta > \frac{r}{q_2}, && \theta > \frac{r-p}{q_2-q_1} \end{aligned} \tag{6}$$

Adding the numerators and denominators of two positive fractions yields a third fraction between the two, so we must have either (A) $\frac{p}{q_1} \le \frac{r}{q_2} \le \frac{r-p}{q_2-q_1}$, or (B) $\frac{p}{q_1} \ge \frac{r}{q_2} \ge \frac{r-p}{q_2-q_1}$. The behavior of consumers depends on which of these configurations they face. The choices that consumers make, as a function of type, are graphed under both Configuration A and Configuration B in Figure 2.

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2 Scotchmer and Maurer also argue that this benchmark cannot be what congress intended for the case of sequential patents, since it leads to the same market outcome as if the patents were not blocking. Presumably, the decision to make the second invention infringing must have some effect. This argument seems incomplete to us, because the decision to make the second product infringing would still have the effect of transferring profit to the first firm, through the lump-sum, for example. This recognizes that the first inventor must be partly credited for the downstream product.

3 We have also considered each of the royalty-based licenses explored below as candidate benchmarks. In each case, however, we have found either a) profits can fall below the monopoly profit for product 1 for a sufficiently small improvement, b) a product is priced over the monopoly price, yielding a loss of both profits and consumer surplus, or c) in some cases, profits are simply equivalent to those enjoyed by one firm with both patents. Because of these pathologies, we continue to believe that our benchmark is the best choice.

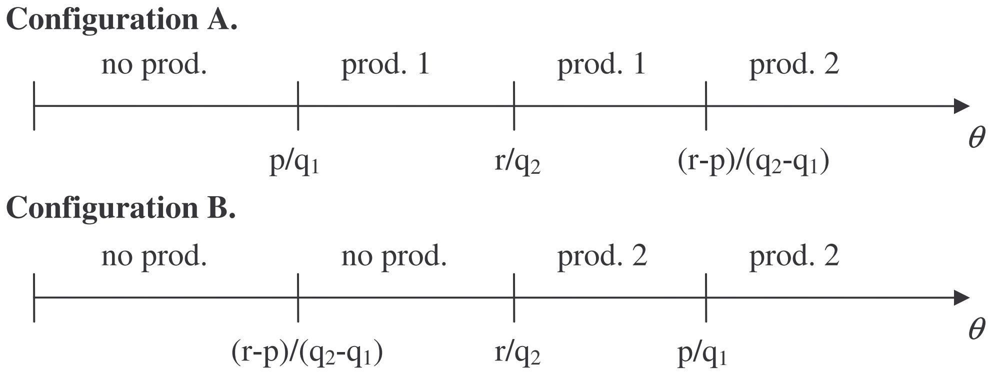

Figure 2

Configuration A is a typical situation in which the price of product 2 is higher than the price of product 1, after adjustment for quality ($\frac{p}{q_1} \leq \frac{r}{q_2}$). In other words, product 1 is a "better deal." This means that as a consumer becomes more quality-sensitive (moves to the right in Figure 2), she will first purchase product 1. The consumer will purchase product 2 only when her sensitivity exceeds the "upgrade price" of $(r-p)/(q_2-q_1)$.

We might call configuration B "inverted," since the higher quality good is a better deal than the lower quality good. As a consumer becomes more quality sensitive, she will first be willing to purchase product 2. At some point, the consumer will see purchasing product 1 as preferable to buying nothing, but at this point, her sensitivity is already higher than the upgrade price, so the consumer will continue purchasing product 2.

Recalling that consumers are distributed in type according to density function *f*, we can integrate to find the profit that the monopolist makes:

$$\pi = \begin{cases} p\int_{p/q_1}^{\frac{r-p}{q_2-q_1}} f(\theta)d\theta + r\int_{\frac{r-p}{q_2-q_1}}^{\infty} f(\theta)d\theta, & Config.\ A \\ r\int_{\frac{r}{q_2}}^{\infty} f(\theta)d\theta, & Config.\ B \end{cases} \tag{7}$$

The next claim shows that the firm will never price discriminate. This is a result of our assumption of zero cost production.

Claim 1. The monopolist with both patents maximizes profits by selling only product 2 at its monopoly price.

We prove this claim in Appendix A. We will label the monopoly price for product 2, r_m. If we employed an alternate assumption that product 2 was more expensive to produce than product 1, we would find that the monopolist would sell some positive amount of product 1, in an attempt to price-discriminate among consumers. On the other hand, our assumption makes it very easy to identify when the firms are producing the optimal level of profit, so the analysis will be much clearer.

We conclude the section by introducing a useful definition. For each license we study, we consider the maximum total profit that can be achieved in any equilibrium.[4] We will call the ratio of this profit to the benchmark profit as the license's *efficiency*. Then a profit neutral license is equivalently a license with efficiency one.

4.2 Lump-Sum License

We now turn to the simplest type of license – a royalty-free license purchased for some lump-sum, or equivalently for some flow payment unrelated to sales. Consumers face two products on the market as before, so their behavior as a function of p and r is identical. However, the profits from selling each product now accrue to separate firms. Ignoring the lump-sum, which is a sunk cost, we write the profits in Configuration A (firm 1 earns zero revenue in configuration B, so an equilibrium will never happen there),

$$\begin{aligned} \pi_1 &= p\int_{p/q_1}^{\frac{r-p}{q_2-q_1}} f(\theta)d\theta \\ \pi_2 &= r\int_{\frac{r-p}{q_2-q_1}}^{\infty} f(\theta)d\theta \end{aligned} \tag{8}$$

Claim 2. In any Nash equilibrium for the lump-sum license, total profits are below the benchmark level. As q_2 approaches q_1, total profits approach zero.

Equivalently, any lump-sum license has efficiency less than one, and the efficiency approaches zero as the quality gap approaches zero. We omit the proof, since we expect the reader will find this result highly intuitive. Without royalties, the firms are in price competition with each other. Firm 1 will always set p/q_1 below r/q_2 in order to generate positive revenues, which already shows profits are suboptimal. As the products become more similar, we approach a game of Bertrand competition with identical products, so we expect profits to approach zero.

We do not want to linger on the poorly performing lump-sum license for long, but it is instructive to consider specific reasons that this license is seldom seen in the wild. They include the following:

1. Before product 2 enters the market, estimates of consumer demand are speculative. Firms will likely disagree on what the expected profits will be, and hence may not be able to agree on a reasonable lump-sum.

[4] The alert reader will notice that we have not placed enough restrictions on F to guarantee that an equilibrium exists for all licenses. In fact, all conditions that we have found to guarantee existence have been messy, so we prefer to omit them. The efficiency of a license can be defined to be zero in the case of no equilibrium. In the example distributions we examine, an equilibrium always exists.

2. The lump-sum license does not allow firms to share the risk associated with the new product. This is costly if firms are risk averse. Firm 2 risks losing money if revenue from product 2 does not cover the lump-sum.
3. Since Firm 2 is in a better position to appraise the demand for product 2, there is a hidden information problem. During negotiation, Firm 2 has a strong incentive to understate the value of its improvement. The firm might not be able to convince firm 1 of what profits it expects without revealing information it may wish to keep secret. This may include trade secrets related to making the downstream product, but it may also include general information about company structure, financial data, and so forth.
4. Finally, such a license may violate antitrust law, since there's no guarantee that the payment will be related to the value of either innovation, violating the principle of derived reward. The Justice Department has specifically suggested that, "it is unlawful for a patentee to insist… that his licensee pay royalties in an amount not reasonably related to the licensee's sales of products covered by the patent."[14]

These issues apply not only to a lump-sum license, but to any license that requires most value to be transferred through fixed fees. Any such license is difficult to negotiate, and may also violate the law.

We lastly note two ways to improve the lump-sum license to achieve optimal profits:

a. Firm 1 contracts to keep product 1 off the market.
b. The firms contractually fix the prices for both products at the monopoly price.

Method a. may seem less invasive, but it only works in our stylized example because optimal profits are achieved by selling only product 2. As we noted above, with richer assumptions, optimal profits will be gained by selling some of both products, so this solution will not work in general.

The second method is brute force price-fixing. It can be checked that fixing just one price will improve total profits, but short of the optimum.

4.3 Flat Royalty License

A more practical license will involve royalties from firm 2 to firm 1. Here, we assume that firm 2 contracts to pay some flat fee, a, to firm 1 for every unit of product 2 that it sells. We first show that such a license is generally not profit neutral. We next discuss why this license has excellent performance for small quality improvements, but is impractical for large quality steps. Finally, we evaluate possible instruments to restore optimal profits.

Claim 3. In any pure strategy Nash equilibrium for the flat royalty license, total profits are below the benchmark level.

Proof. We will demonstrate that firm 1 sells some of product 1 in any equilibrium. This implies that total profits are sub-optimal as before. Firm profits are,

	Configuration A	Configuration B
π_1	$p\int_{p/q_1}^{\infty} f(\theta)d\theta + (a-p)\int_{\frac{r-p}{q_2-q_1}}^{\infty} f(\theta)d\theta$	$a\int_{\frac{r}{q_2}}^{\infty} f(\theta)d\theta$
π_2	$(r-a)\int_{\frac{r-p}{q_2-q_1}}^{\infty} f(\theta)d\theta$	$(r-a)\int_{\frac{r}{q_2}}^{\infty} f(\theta)d\theta$

(9)

Consider how firm 1's profit changes with respect to *p*:

$$\frac{\partial \pi_1}{\partial p} = \begin{cases} -\frac{p}{q_1} f\left(\frac{p}{q_1}\right) - F\left(\frac{p}{q_1}\right) + \frac{(a-p)}{q_2-q_1} f\left(\frac{r-p}{q_2-q_1}\right) + F\left(\frac{r-p}{q_2-q_1}\right), & \frac{p}{q_1} < \frac{r}{q_2} \\ 0, & \frac{p}{q_1} > \frac{r}{q_2} \end{cases} \quad (10)$$

We take the limit as we approach the discontinuity from the interesting direction:

$$\text{as } \frac{p}{q_1} \xrightarrow{-} \frac{r}{q_2}, \quad \frac{\partial \pi_1}{\partial p} \to -\frac{r-a}{q_2-q_1} f\left(\frac{r-p}{q_2-q_1}\right) \quad (11)$$

In equilibrium, we expect that firm 2 will choose a positive profit margin, $r-a>0$, which implies that the above limit is negative. This means that firm 1 could decrease *p* in order to sell a small amount of product 1, thereby increasing profits. Therefore, in a pure strategy equilibrium, there must be some positive amount of product 1 sold, which completes the proof.

The above claim shows that any flat rate license has efficiency strictly less than one. Although this is interesting, we want to know exactly how far below one the efficiency lies. If the license is close enough to optimal, price-fixing will not significantly improve firm profits, so it is probably safe to forbid it. Such a policy might present benefits not encompassed by this model, including enhanced protection against sham licenses, or lower enforcement costs.

To begin, suppose that we choose the flat rate specifically to maximize license efficiency. As the next claim shows, we find that profits approach their optimal levels in the limiting cases of very small and very large quality improvements.

Claim 4. For any *e*<1, there exists an *l*>0 and an *L*>0 such that when $\frac{q_2-q_1}{q_1} < l$, or $\frac{q_2-q_1}{q_1} > L$, the flat rate a can be chosen so that any pure strategy equilibrium for the flat rate license has efficiency greater than *e*.

The proof is in Appendix B. Loosely speaking, the best possible flat rate license is near optimal for both very small and very large quality improvements. This is clearly visible in simulation results presented in Figure 3. For this plot, q_1 has been fixed at 1, and q_2 varies along the horizontal axis. Three different consumer distributions are presented: a uniform distribution, a Gaussian with mean 0 and standard deviation 1, and an exponential, e^{-x}. For each value of q_2, we compute the maximum efficiency of a flat rate license for each distribution and plot this on the vertical axis. The resulting curve seems to be robust; we generated very similar plots for a variety of other Uniform, Gaussian, and Exponential distributions.

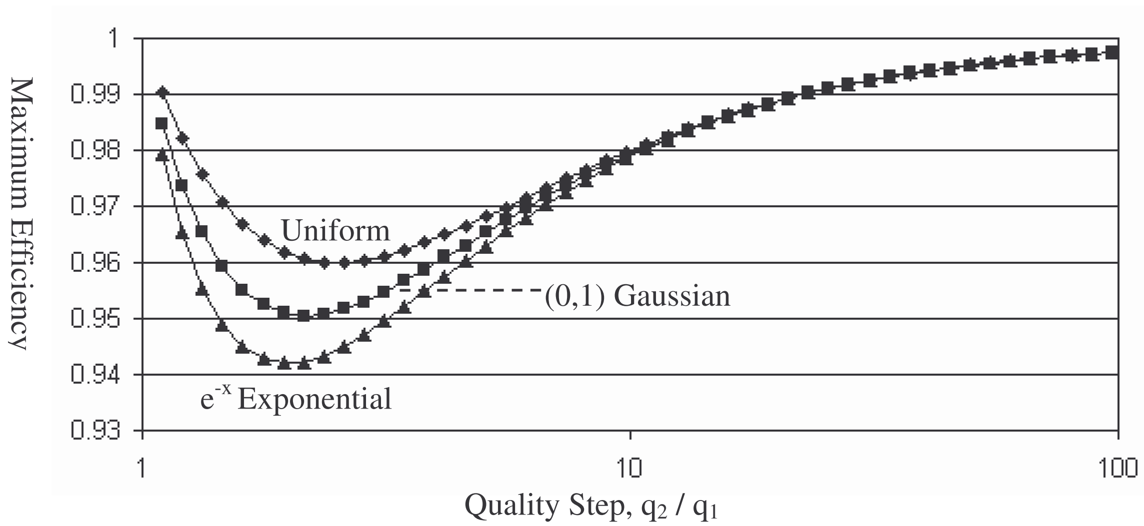

Figure 3: Efficiency of the optimal flat rate license for a variety of consumer distributions

Surprisingly, we always found a flat rate license with efficiency over 94%. Remember, though, that this is the maximum efficiency possible, and selecting the royalty imperfectly will result in a lower efficiency. In particular, the appropriate royalty may be impractical for large q_2. Figure 4 shows what the optimizing royalty is, plotted for comparison against the equilibrium prices. What is striking is how the optimal rate remains very low (near 1), even while firm 2's price increases dramatically towards the right.

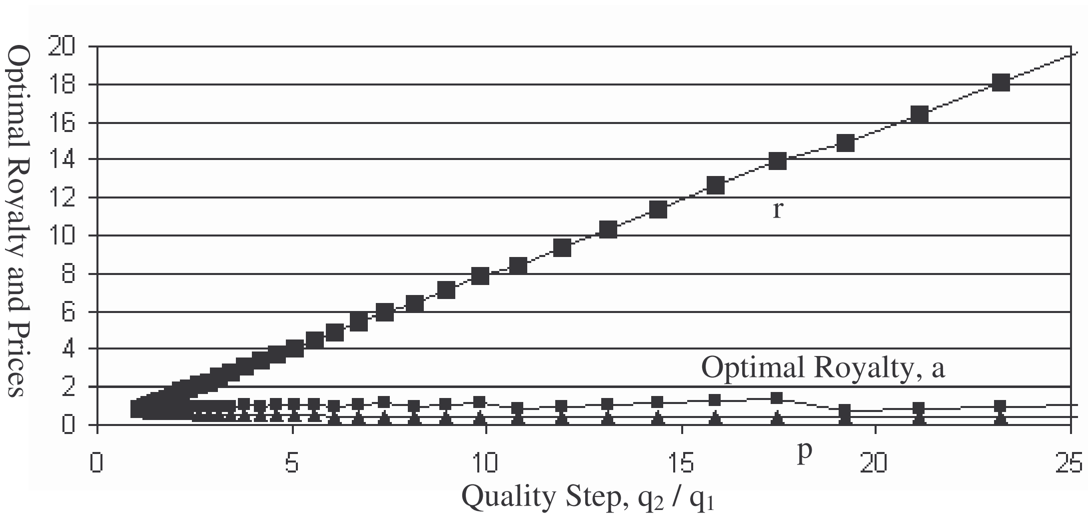

Figure 4: Optimal flat rate royalty for a (0,1) Gaussian Distribution, compared to equilibrium prices

Intuitively, as the quality step grows large, product 1 becomes largely irrelevant from the perspective of total profits -- product 2 has the potential to generate far more revenue. Maximizing total profits becomes a matter of making sure *r* falls near the monopoly price, and the way to do this is by setting a low royalty. Setting a royalty on the order of the monopoly price would result in product 2 being double marginalized, lowering profit substantially (while also reducing consumer surplus).

For large q_2, under the optimal license, firm 1 receives little of the value of the downstream product through royalties. Presumably, the firms will negotiate to transfer more of the value to firm 1 through fixed fees. Unfortunately, when the fixed fees dwarf the royalties, the license becomes difficult to agree upon for the same reasons listed for the lump-sum license. Because of this, we believe the optimal license is impractical for large quality steps.

With this in mind, we turn our attention to possible instruments that can improve the performance of the flat rate license.

Claim 5. Any one of the following modifications, when added to the flat rate license, results in optimal profits.

a. Contract to fix p at p_m, the monopoly price for q_1.
b. Contract to fix r at r_m, the monopoly price for q_2.
c. Contractually set both $p = p_m$ and $r = r_m$.
d. Allow firm 1 to pay firm 2 a royalty on sales of product 1.

Proof: (a) With p fixed at p_m, the firms can set the royalty $a = p = p_m$. Firm 2 then experiences profits,

$$\pi_2 = \begin{cases} (r - p_m)\int_{\frac{r-p_m}{q_2-q_1}}^{\infty} f(\theta)d\theta, & \frac{r}{q_2} \geq \frac{p_m}{q_1} \\ (r - p_m)\int_{\frac{r}{q_2}}^{\infty} f(\theta)d\theta, & \frac{r}{q_2} < \frac{p_m}{q_1} \end{cases} \tag{12}$$

In Configuration A (top formula), firm 2's profit is the same as a monopolist earns from selling upgrades at price $r - p_m$ (compare to the last term in equation 18, Appendix A). From this analogy, we know the top formula reaches its maximum when $r - p_m$ equals the monopoly upgrade price (the monopoly price for quality $q_2 - q_1$). Since p is already at the monopoly price, this occurs at $r = r_m$, on the boundary between configurations.

The bottom formula holds in Configuration B, which can be characterized by the condition, $\frac{r}{q_2} > \frac{r - p_m}{q_2 - q_1}$. Inspection shows that the value of the bottom formula is less than the value of the top formula in this range. Since the top formula has a unique maximum at r_m, this profit will not be exceeded for lower values of r. Hence, r_m is firm 2's best response, and optimal profits result.

(b) With r fixed at r_m, the firms can set $a = r$. This means that firm 2 transfers all of its revenue to firm 1. Firm 1's profits are then equal to the total profits, and firm 1 can then set $p = p_m$ and receive same profit as a monopolist holding both inventions (and firm 1 can do no better since this is the theoretical maximum profit attainable).

(c) Fixing both prices is trivially still a way to optimize profits.

(d) Finally, we note that cross royalties can yield the optimal configuration. In our stylized example, any sufficiently large royalty from firm 1 to firm 2 will work to discourage firm 1 from selling any of product 1, so firm 1 can set p very high to yield an optimal equilibrium. In a more general model in which price discrimination is necessary to yield the optimum, more specific royalties are required to enforce proper behavior.

Claim 5 shows that price fixing is one way to achieve perfect efficiency in a flat-rate license. Perhaps surprisingly, the license only has to fix the price of one good, not both. Unfortunately, fixing r transfers all sales revenue to firm 1, so firm 2 must be compensated entirely by fixed fees. As before, we expect that such a license is impractical, or at least costly to negotiate.

Fixing p is somewhat more promising. When the upstream good's price is fixed, we find that profits are optimized at $a = p$, meaning firm 1 earns the same payoff when either product is sold. In a sense, this royalty is a natural choice. It corresponds to the situation in which firm 2 buys units of product 1 directly from firm 1 at the normal market price and then upgrades them, creating units of product 2, which it then sells to end-consumers. For a concrete example, firm 1 might produce operating systems, which firm 2 buys and then outfits with text editing software, before reselling them at a higher price. Of course, firm 2 probably doesn't need a license to do this, since the first sale doctrine protects firm 2's right to modify and resell particular embodiments of firm 1's patent that it has legally purchased. Without the license to fix p, however, merely setting $a = p$ does not result in profit neutrality. If the quality gap is large, the downstream product simply becomes double-marginalized.

Fixing the price of the upstream good, moreover, does not correct the impracticality we observed above – for large q_2, firm 1's royalties are small compared to r, and most value must be transferred via fixed fees. Nevertheless, we believe that such price-fixing may be applied to improve the efficiency of the license for intermediate quality steps.

Of course, fixing both prices yields optimal profits. When the quality gap is small to intermediate, such a restriction apparently fails the principle of minimalism. For small quality gaps, the efficiency is near 1 without any price fixing. For intermediate quality gaps, it is sufficient to fix the price of just one firm, and the other firm's behavior can be controlled by royalties. What about large quality gaps? From all we have seen so far, fixing both prices may be the only feasible way to achieve profit neutrality for large q_2. However, we will later see that a percentage rate license is a superior solution, so we argue that fixing both prices fails the principle of minimalism in all cases.

Claim 5 finally shows that cross royalties can alternately be used to achieve profit neutrality, but such a solution probably violates the derived reward principle. Firm 2 did not invent product 1, but would seem to be deriving a reward from product 1 sales. It is worth pointing out that cross royalties may be mathematically subtracted from each other to yield an equivalent single royalty from firm 2 to firm 1 that varies with firm 1's price and sales. If cross royalties are forbidden, royalties that are economically equivalent to such royalties should presumably also be forbidden.

4.4 Percentage Rate License

We now turn our attention to our final example license: one that computes royalties as a percentage of sales revenue. Firms may prefer this license for a variety of reasons. First, firm 1 is protected against a firm 2 that tries to pretend that its improvement is small, in order to secure a small royalty but sell at a high price. Secondly, firm 1 may require less detailed information about product 2 before agreeing. This in turn protects firm 2 from a firm 1 that wants to steal the improvement during negotiations. Finally, the percentage rate may be constrained by cultural expectations. That is, firms may enter negotiations with a preconceived idea of what a "reasonable" royalty is, which may reflect agreements that similar firms reached in the past. This would result in lower transaction costs.

To analyze this license, we specify that firm 2 agrees to transfer a certain percentage of its revenue from selling product 2, α, to firm 1. As we did for the flat rate license, we first demonstrate that the percentage rate license is not profit neutral. We then demonstrate that the license performs well for large quality steps, but is dramatically suboptimal when the quality step is small (the reverse of the flat rate license). Finally, we consider instruments that can be added to the license to optimize profits. These instruments will be analogous to the ones we discovered for the flat rate license.

Claim 6. For α<1, in any pure strategy Nash equilibrium for the percentage rate license, total profits are below the benchmark level.

Proof: As before, it suffices to show that firm 1 sells some positive amount of product 1 in equilibrium. Firm profits are now given by,

	Configuration A	Configuration B
π_1	$p\int_{p/q_1}^{\frac{r-p}{q_2-q_1}} f(\theta)d\theta + \alpha r \int_{\frac{r-p}{q_2-q_1}}^{\infty} f(\theta)d\theta$	$\alpha r \int_{\frac{r}{q_2}}^{\infty} f(\theta)d\theta$
π_2	$(1-\alpha)r\int_{\frac{r-p}{q_2-q_1}}^{\infty} f(\theta)d\theta$	$(1-\alpha)r\int_{\frac{r}{q_2}}^{\infty} f(\theta)d\theta$

(13)

From which the effect of p on firm 1's profit is,

$$\frac{\partial \pi_1}{\partial p} = \begin{cases} -\frac{p}{q_2-q_1} f\left(\frac{r-p}{q_2-q_1}\right) - \frac{p}{q_1} f\left(\frac{p}{q_1}\right) + \alpha \frac{r}{q_2-q_1} f\left(\frac{r-p}{q_2-q_1}\right) \\ \quad + F\left(\frac{r-p}{q_2-q_1}\right) - F\left(\frac{p}{q_1}\right), & \frac{p}{q_1} < \frac{r}{q_2} \\ 0, & \frac{p}{q_1} > \frac{r}{q_2} \end{cases} \tag{14}$$

We find that the limit as we approach the discontinuity from the interesting side is negative, just as for the flat rate license:

$$\text{as } \frac{p}{q_1} \xrightarrow{-} \frac{r}{q_2}, \quad \frac{\partial \pi_1}{\partial p} \to -(1-\alpha)\frac{r}{q_2-q_1} f\left(\frac{r-p}{q_2-q_1}\right) < 0 \tag{15}$$

The inequality is strict, because we know $f\left(\frac{r-p}{q_2-q_1}\right) > 0$. If this were not the case, firm 2 could raise r without losing customers, contradicting our assumption of equilibrium. Relation 15 implies that for any strategy point in which only product 2 is sold, firm 1 can lower p to sell a little bit of product 1 and enhance profits. Thus, any equilibrium solution has some of product 1 being sold, and this completes the proof.

Similarly to the previous section, we now know that the percentage rate license has efficiency less than one, but we really want a better picture of how the efficiency depends on product qualities. The next claim shows that the efficiency approaches 1 as the quality step grows large, but approaches zero as the quality step approaches zero:

Claim 7. a. For any *e*>0, there exists an *l*>0 such that for $\frac{q_2 - q_1}{q_1} < l$, any pure strategy equilibrium for the percentage rate license has efficiency less than *e*.

b. For any *e*<1, there exists an *L*>0 such that for $\frac{q_2 - q_1}{q_1} > L$, any pure strategy equilibrium for the percentage rate license has efficiency greater than *e*.

This claim is proved in Appendix C. Figure 5 shows simulation results for a (0,1) Gaussian distribution, clearly demonstrating the limiting behavior. Nearly identical graphs resulted for the other distributions we studied.

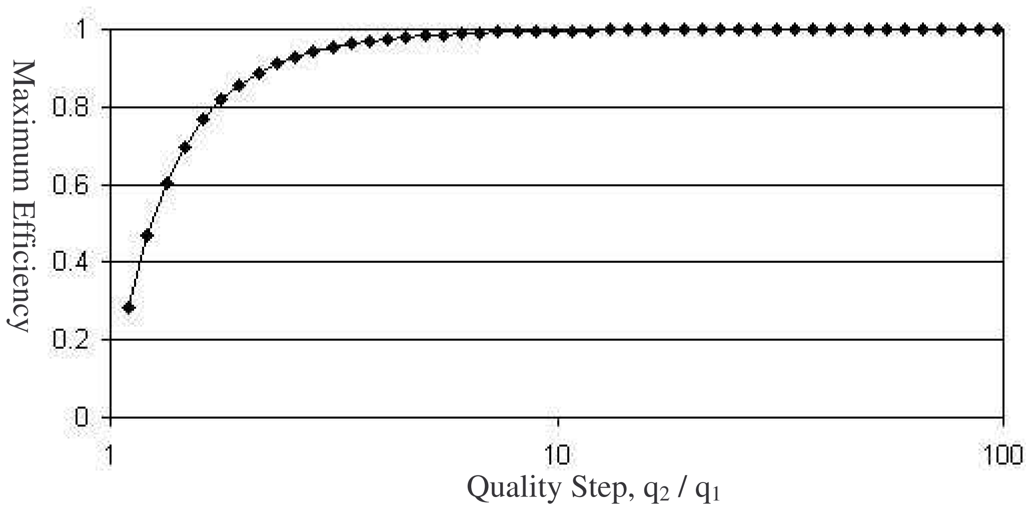

Figure 5: Efficiency of the optimal percentage rate license, (0,1) Gaussian distribution

Unlike the corresponding claim for the flat rate license, the limiting behavior here does not depend on a specific choice of royalty. Any percentage rate results in near perfect efficiency for large quality steps. Because of this, we remark that the percentage rate license performs well for large quality steps. This explains why, in the previous section, we rejected a flat rate license with both prices fixed for large quality steps. The percentage rate license is clearly more minimal.

The problem with the percentage rate license for small q_2 is particularly acute. As the quality gap shrinks, profits without price-fixing approach zero, so the total profit will eventually fall below the profit firm 1 can enjoy from selling product 1 alone. When this happens, firm 1 will prefer to exercise its patent to block firm 2 from the market entirely. Of course, without specifying a particular game of entry, we expect that firm 2 would have predicted this outcome, and not invested in the first place. Hence, the percentage rate license has the pathological effect of destroying all incentive for firm 2 to invest in small quality improvements. We next evaluate possible ways to correct this problem and make the percentage rate license profit neutral.

Claim 8. Any one of the following modifications, when added to the percentage rate license, results in optimal profits.

a. Fix $p = p_m$.
b. Fix $r = r_m$ (if α can equal 1)
c. Contractually set both $p = p_m$ and $r = r_m$.
d. Allow firm 1 to pay firm 2 a percentage royalty on sales of product 1.

Proof: (a) With $p = p_m$, firm 2's profit is given by:

$$\pi_2 = \begin{cases} (1-\alpha)r\int_{\frac{r-p_m}{q_2-q_1}}^{\infty} f(\theta)d\theta, & \frac{r}{q_2} \geq \frac{p_m}{q_1} \\ (1-\alpha)r\int_{\frac{r}{q_2}}^{\infty} f(\theta)d\theta, & \frac{r}{q_2} < \frac{p_m}{q_1} \end{cases} \tag{16}$$

In Configuration B (bottom formula), firm 2's profit is simply a constant fraction of what a monopolist earns selling just product 2. This is maximized when r is set to r_m. Since p is already at the monopoly price, this point lies on the boundary between configurations.

On the other hand, in Configuration A (top formula), we have the condition $\frac{r}{q_2} \leq \frac{r-p_m}{q_2-q_1}$, which implies that the top formula is no greater than the bottom. Since the bottom formula has a unique maximum at r_m, this profit is never exceeded for greater values of r. This means that r_m is firm 2's best response, so the optimal profits result. Note that this result does not depend on what the percentage rate is.

(b) If r is fixed at r_m, and the firms set α=1, the second firm transfers all of its revenue to the first firm. This means that the first firm's profits equal the total profits, so they are maximized when p=p_m. Hence, optimal profits result, but the license is impractical since firm 2 must be compensated solely by fixed fees.

(c) Just as before, fixing both prices is a possible solution. Again, however, this instrument seems to fail the principle of minimalism, since only one price needs to be fixed in order to achieve maximum profits.

(d) Suppose that firm 1 agrees to pay firm 2 a fraction, $1-\alpha$, of its sales revenue. Profits are then:

$$\begin{aligned} \pi_1 &= \alpha p\int_{p/q_1}^{\frac{r-p}{q_2-q_1}} f(\theta)d\theta + \alpha r\int_{\frac{r-p}{q_2-q_1}}^{\infty} f(\theta)d\theta \\ \pi_2 &= (1-\alpha)p\int_{p/q_1}^{\frac{r-p}{q_2-q_1}} f(\theta)d\theta + (1-\alpha)r\int_{\frac{r-p}{q_2-q_1}}^{\infty} f(\theta)d\theta \end{aligned} \tag{17}$$

From which we can see that $(1-\alpha)\pi_1 = \alpha\pi_2$. Each firm's profit is now a constant fraction of the other firm's profit, and also a constant fraction of the total profit. Since their incentives are aligned, by maximizing their own profits, both firms will also maximize total profits yielding the optimum. One must remember, however, that this method probably fails the principle of derived reward, since firm 2 receives payment for the sale of product 1, which it did not invent.

Analogous to the case of the flat rate license, we find that price fixing can make the percentage rate license profit neutral. Fixing r is impractical, just as for the flat rate license, since firm 2 must be compensated entirely by fixed fees. It seems quite advantageous to fix $p = p_m$, since the royalty can then be set to anything, dividing profits however firms agree. If the quality gap is large, this fails the principle of minimalism, since profits are near optimal without any price fixing. For small and intermediate quality gaps, on the other hand, our model predicts that such price fixing should be permitted.

As for the flat rate license, cross royalties yield perfect efficiency, but likely fail the principle of derived reward.

5. Application to Policy

We summarize the results of the previous section as follows:

1. Lump-sum, flat rate, and percentage rate licenses are not profit neutral. However, flat rate licenses are close to optimal for small quality steps, while percentage rate licenses are close to optimal for large quality steps. Flat rate licenses are impractical for large quality steps, while percentage rate licenses suffer from poor performance for small quality steps.
2. Price fixing can be applied to make a license profit neutral. In general, only one price has to be fixed, while the other can be controlled by royalties. Cross royalties may also be applied, though these likely violate the principle of derived reward.

We observe that the arguments for and against price-fixing depend on the quality gap between products. In order for a court or regulator to apply our model, a first step is to assess whether the quality gap is small, intermediate, or large. The analysis differs in each case:

Small quality step. When the products are very similar, a percentage rate license has dramatically poor performance. A flat rate license has nearly optimal performance. Unfortunately, firms may find it easier to negotiate a percentage rate license when they don't have full information. Recall that firm 2 is in a better position to judge q_2, and has an incentive to under-report its valuation to firm 1, hoping for a lower flat rate. If firm 2 has to reveal sensitive information to make firm 1 believe its valuation, firms may be unable to agree on what the quality is. It then becomes difficult to settle on a single flat royalty, and firms may prefer a percentage rate. Even if firm 2 doesn't have hidden information, firms will prefer the more flexible percentage rate when consumer demand is unknown or volatile.

We may also speculate that few licenses act purely like a flat-royalty license in a dynamic setting. If the royalty rate can be reset at certain intervals, or if the license can be renewed with a different royalty, the licensee may expect the licensor to base future royalties on present-day prices, and the resulting behavior may combine aspects of both the flat-rate and percentage-rate licenses. We defer these considerations to future studies.

In judging a scenario with a small quality step, the crucial question becomes how much hidden information there is. Does firm 2 have valuable trade secrets, and is consumer demand predictable? If there is not much hidden information, it appears safe to forbid price-fixing, since firms can negotiate a nearly-optimal flat rate license. Otherwise, firms should be allowed to fix the price of one good. They can then select a percentage rate license, with the p fixed, to receive maximum profits.

Intermediate quality step. For intermediate quality steps, both percentage rate and flat rate licenses have reasonable, but not perfect efficiency, making it difficult to form a policy recommendation. Towards the small-to-intermediate quality step range, the percentage rate license still has poor performance, so we must ask, as above, whether firms can agree to a flat rate. If the answer is no, price-fixing should be permitted. As the quality step increases in this range, both types of licenses have high, but not perfect efficiency. A tradeoff exists between rewarding firms with the last fraction of optimal profits, and the extra benefits of forbidding price-fixing. We suggest that policy makers could establish a cutoff efficiency to guide this decision. If license efficiency falls below the cutoff, price-fixing can be permitted, but only on one product. Our model is not positioned to guide the choice of this cutoff.

Large quality step. When the quality step is large, a percentage rate license is a simple and easy to negotiate choice with nearly optimal efficiency. In this case, price-fixing should be forbidden.

In light of this discussion, we return to consider the *General Electric* decision. As we suggested earlier, firm 2 does not necessarily protect its improvement with a patent – subject to certain caveats, it may hold it as a trade secret. This more accurately depicts the problem posed by *General Electric*, since Westinghouse did not take out a patent on its version of the light bulb. Prior literature typically assumes that the bulbs manufactured by each company were perfect substitutes. This is also suggested by the fact that General Electric and Westinghouse decided to price them identically. We therefore consider our finding for small quality gaps.

Technically, General Electric did not fix either price at the monopoly price in its contract, but rather locked one price to the other, $r = p$. If we assume that $q_2 = q_1$, this also fixes $r / q_2 = p / q_1$, and it is easy to verify that firm 1 will choose $p = p_m$, which maximizes total profits, so such a license is profit neutral in our model.

For small quality gaps, we ask how much information is hidden from the firms. If the quality of Westinghouse's light bulbs was secret, or if the demand for light bulbs was sufficiently unknown or volatile, the firms may have been unable to agree on a flat rate. In this case, price fixing should be permitted to prevent the erosion of profits seen for a percentage rate license. The Supreme Court's argument, quoted earlier, that 'Yes, you may make and sell articles under my patent but not so as to destroy the profit that I wish to obtain by making them and selling them myself,' [12] seems well targeted to this case.

In the case of *Line Material*, we find it difficult to isolate a clear policy lesson because of the complicated nature of the licensing scheme. Certainly, we suspect that Line's gravity-driven fuse was a major improvement over its predecessor. Our model predicts that when the quality gap is large, a percentage rate license can sustain nearly optimal profit levels, without price-fixing. This is also suggested by the fact that Southern States voluntarily stopped producing the upstream product after cross-licensing with Line. If marginal costs of production are similar enough for the two products, this is exactly what must happen to attain the optimal profits, just as we observed in our model.

Contrary to our assumptions, however, it was not the price of the inferior fuse or the price of all the superior fuses that was fixed. Rather, Line specifically fixed the price of third-party manufacturers of the superior fuse. We must then ask, just as for *General Electric*, whether the fuses that Line and the other manufacturers produced were very similar, and whether there was hidden information. If the fuses were indistinguishable to consumers, and if market conditions were sufficiently known so that a flat-rate license could be negotiated, we expect that price-fixing was unnecessary to maintain optimal profits, and the Court was correct to forbid it.

6. Discussion

Our results form a complement to those already discovered by Scotchmer and Maurer [7]. Whereas they find that price-fixing may be needed to distribute manufacturing efficiently, we find that price-fixing may be needed to stop the erosion of profits that results from competition between generations of an invention. In the extreme, this erosion can reduce total profits to zero, destroying all incentive to create particular improvements. For large quality gaps between product generations, the percentage rate license is best at containing this erosion. The best flat rate license works in theory, but transfers too little value to firm 1. For small quality gaps, a flat rate license is a better way to contain the erosion of profits. Unfortunately, we give reasons to believe that a flat royalty may not be possible when there is hidden information. Our results suggest that price-fixing should be permitted for small quality gaps in which flat rate licensing is infeasible, but not for large quality gaps. For intermediate quality gaps, a policy must be selected to balance optimizing profits against the benefits of forbidding price-fixing.

In this study, our simplifying assumptions, particularly of zero-cost production, served to make the analysis tractable and clear, but they also obscured certain issues. A richer model would highlight more complex firm behavior, such as price-discriminating to attain optimal profits. There is also the possibility of specifically dynamic concerns, for example, what happens when demand is volatile over time.

It is important to remember that price-fixing is only one possible license element that can yield profit neutrality. We showed that profits can also be optimized using a system of cross-royalties, though this seems to violate the principle of derived reward. If new instruments are discovered that generate profit neutrality, and these instruments are less susceptible to abuse than price-fixing, price-fixing may again be deemed to violate minimalism in favor of these new measures.

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Appendix

A. Proof of Claim 1

Since configuration B profits are unaffected by *p*, any strategy that falls in configuration B yields the same profits that it would if *p* were reset to $p = r\frac{q_1}{q_2}$, on the boundary between configurations. Conveniently, this new strategy is describable by the Configuration A profit function. This means that we only have to find the profit maximizing point in Configuration A.

We show that the Configuration A profits are maximized at the boundary between configurations. Configuration A profit can be rewritten,

$$\text{Config. A: } \pi = p\int_{p/q_1}^{\infty} f(\theta)d\theta + (r-p)\int_{\frac{r-p}{q_2-q_1}}^{\infty} f(\theta)d\theta \tag{18}$$

In other words, total profits are the sum of profits from selling product 1, plus the independent profits from upgrading consumers with product 1 to product 2. From our IHR assumption, both components of the profit have a unique maximum value, which falls at the monopoly price-quality ratio, so,

$$\frac{p}{q_1} = \frac{r-p}{q_2-q_1} \tag{19}$$

This point lies on the boundary between configurations, so the profits at this point are correctly described by 18, as required. Examining Configuration B profits shows that the product will be sold at the monopoly price for a product of quality q_2.

B. Proof of Claim 4

First we consider the limit as the quality step grows small. Then, we consider the limit as the quality step grows large.

Step 1: For any e<1, there exists an l>0 such that for $\frac{q_2 - q_1}{q_1} < l$, the flat rate a can be chosen so that any pure strategy equilibrium for the flat rate license has efficiency greater than e.

Specifically, we fix $a = p_m$, the monopoly price for quality q_1. Suppose that (p, r) is an equilibrium for this rate. First, note that $r > a$, to give firm 2 positive profit.

Next, since all equilibria are in Configuration A, firm 1's first order condition is,

$$\frac{\partial \pi_1}{\partial p} = -\frac{p}{q_1} f\left(\frac{p}{q_1}\right) + 1 - F\left(\frac{p}{q_1}\right) + \frac{a-p}{q_2-q_1} f\left(\frac{r-p}{q_2-q_1}\right) - \left[1 - F\left(\frac{r-p}{q_2-q_1}\right)\right] = 0 \quad (20)$$

The first three terms can be interpreted as the marginal profit for a monopolist selling just product 1. Since *a* is the monopoly price, this marginal profit is negative for $p > a$. It is easy to check that the last two terms are also negative for $p > a$, so we know $p \le a$.

Since *p* is no greater than the monopoly price, the first three terms of 20 must sum to be non-negative, meaning the last two terms must sum to be non-positive:

$$\frac{a-p}{q_2-q_1} f\left(\frac{r-p}{q_2-q_1}\right) - \left[1 - F\left(\frac{r-p}{q_2-q_1}\right)\right] \le 0, \quad (21)$$

or,

$$\frac{q_2-q_1}{a-p} \ge h\left(\frac{r-p}{q_2-q_1}\right) \ge h\left(\frac{a-p}{q_2-q_1}\right), \quad (22)$$

where *h* is the hazard rate of *F*, and the last inequality follows since *h* is increasing. Letting $x = \frac{a-p}{q_2-q_1}$, we have,

$$\frac{1}{x} \ge h(x) \quad (23)$$

The left hand side is decreasing, so equality must happen at exactly once point. Let δ be the point such that $1/\delta = h(\delta)$. Then the last relation can be rewritten, $x \le \delta$, or,

$$\frac{a-p}{q_2-q_1} \le \delta. \quad (24)$$

or,

$$\frac{a}{q_1} - \frac{p}{q_1} \le \delta \frac{q_2-q_1}{q_1} \quad (25)$$

Hence, choosing *l* to bound the right hand side appropriately, p/q_1 can be made arbitrarily close to the monopoly price-quality ratio.

Finally, we use the first order condition for firm 2:

$$\frac{\partial \pi_2}{\partial r} = -\frac{r-a}{q_2-q_1} f\left(\frac{r-p}{q_2-q_1}\right) + 1 - F\left(\frac{r-p}{q_2-q_1}\right) = 0, \tag{26}$$

or,

$$\frac{q_2-q_1}{r-a} = h\left(\frac{r-p}{q_2-q_1}\right) \geq h\left(\frac{r-a}{q_2-q_1}\right). \tag{27}$$

By similar argument as above, we find,

$$\frac{r}{q_2} - \frac{a}{q_2} \leq \delta \frac{q_2-q_1}{q_2}, \tag{28}$$

So r/q_2 can be made arbitrarily close to a/q_2 by appropriate choice of bound, *l*, and a/q_2 can be made arbitrarily close to a/q_1, also by appropriate choice of *l*. Hence *l* can be chosen small enough to make r/q_2 as close to the monopoly price quality ratio as desired.

Since *l* can be chosen to make both prices arbitrarily close to the monopoly price-quality ratio, the efficiency of any equilibrium can be guaranteed to be as close to 1 as desired.

Step 2: For any *e*<1, there exists an *L*>0 such that for $\frac{q_2-q_1}{q_1} > L$, the flat rate, a, can be chosen so that any pure strategy equilibrium for the flat rate license has efficiency greater than *e*.

We select $a = 0$, and recalling that all equilibria are in Configuration A, firm 2's first order condition becomes,

$$\frac{\partial \pi_2}{\partial r} = \frac{-r}{q_2-q_1} f\left(\frac{r-p}{q_2-q_1}\right) + 1 - F\left(\frac{r-p}{q_2-q_1}\right) = 0 \tag{29}$$

or

$$h\left(\frac{r-p}{q_2-q_1}\right) = \frac{q_2-q_1}{r} \tag{30}$$

As $\frac{q_2-q_1}{q_1}$, and hence q_2/q_1, grows without bound, the right hand side approaches q_2/r, the left hand side approaches $h(r/q_2)$. Viewing both sides as functions of r/q_2, it may be shown that the intersection approaches the intersection of the limiting functions,

$$\frac{q_2}{r} = h\left(\frac{r}{q_2}\right). \tag{31}$$

This is not true of functions in general, but holds in this case since the intersection is in the interior of the domain, and the functions actually cross. Expanding *h*, it is easy to see that 31 defines the monopoly price-quality ratio.

Hence, as *L* grows large, r / q_2 approaches the monopoly price-quality ratio, which implies that $\frac{r - p}{q_2 - q_1}$ also approaches the monopoly ratio. This implies that the efficiency of the license approaches 1.

C. Proof of Claim 7

a. It is sufficient to show that *l* can be chosen to make p / q_1 and r / q_2 arbitrarily close to zero in any equilibrium.

First recall that any equilibrium lies in configuration A. Firm 2's first order condition reduces to equation 30, above. Since $\frac{r - p}{q_2 - q_1} > \frac{r}{q_2}$, and *h* is increasing by assumption, we have,

$$h\left(\frac{r}{q_2}\right) \leq \frac{q_2 - q_1}{r} = \frac{(q_2 - q_1)/ q_2}{r / q_2} \tag{32}$$

Consider both sides of this relation as functions of r / q_2. On the left, h is an increasing function. On the right, we have a decreasing hyperbola. The two curves must intersect once, say at *x*, and the inequality implies that $r / q_2 \leq x$. Decreasing the numerator of the hyperbola shifts *x* to the left. Furthermore, we can choose an upper bound on the numerator, *l*, to ensure that *x* lies as close to zero as desired. Since $p / q_1 < r / q_2 \leq x$, the claim follows.

b. Firm 2's choice of *r* is again governed by relation 30. The proof then proceeds identically to step 2 of Claim 4.