The Choreography of Conceptual Development in Computer Supported Instructional Environments

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ABSTRACT

A key affordance of computational learning environments is that they can be designed to instantiate specific rules and causal relationships between user inputs and perceptual output. In this sense, designers can attempt to engineer specific trajectories of learners' conceptual development by devising situations they believe will help learners construct a particular concept/scheme. We revisit and elaborate upon Papert's notion of 'body syntonicity' and present a three-parameter model of interaction to describe how the interplay between a learner's prior knowledge, immediate perceptions, and goals embedded into the instructional situation contributes to the emergence of new conceptual schemes. We retrospectively apply this model to prior works and to a current instructional design that combines mathematics learning with physical exercise.

Categories and Subject Descriptors

H.5.2. [**User Interfaces**]

General Terms

Theory, Human Factors, Design

Keywords

Educational technology, learning theory, embodied interaction, embodied cognition, mathematics education, design-based research, physical activity

1. INTRODUCTION

Leveraging learners' prior knowledge, perception, activity, and environment to support their mathematical conceptual development—can be traced back through antiquity. Seymour Papert can be largely credited with the vision of connecting computational experiences with children's knowledge and understanding of themselves [20]. We revisit and elaborate upon Papert's notion of 'body syntonicity' [21] and present a three-parameter model of interaction to describe the interplay between a learner's prior knowledge, immediate perceptions, and the goals embedded into an instructional activity contributes to the emergence of new conceptual schemes.

Motivating this work in part is the observation that an instructional activity may be designed so as to purposefully and productively *evoke* learners' existing funds of knowledge, and/or to *enact* situated experiences that support a learner's development of specific concepts/schemes [7]. The unit of analysis that will inform examination is the relationship between learners' schemes and the situations that give rise to them [28].

This work is intended to inform an analytical framework for understanding how computer-supported interactions can be choreographed to support the learning of conceptual content matter. Specifically, we wish to examine how learners' physical actions enacted in synchrony with computational feedback can influence the developmental trajectories of learners' cognitive schemes. The instructional designs presented herein are influenced in part by theoretical claims that cognition, in general [3], and mathematical reasoning, in particular [15,18], are grounded and tacitly instantiated in real-world experiences. Therefore, explicit consideration is given to exploring how learners' multi-modal perceptions and embodied interactions help mediate and modulate their development of mathematical concepts [2].

We contextualize our views in light of prior design work, and present a proof-of-concept design that attempts to leverage the body as a tangible user interface to support the learning of mathematics through physical exercise.

2. BACKGROUND AND THEORETICAL FRAMEWORK

2.1 Body Syntonicity

Papert [21] first introduced the construct *body syntonicity* to characterize how an instructional design can be "related to children's sense and knowledge about their own bodies" (p. 63). A disciple of the developmental psychologist Jean Piaget, Papert subscribed to the epistemological position that knowledge is constructed by and through experience in the world. He argued that programming languages such as LOGO allowed learners to ground their mathematical understandings in their own subjective phenomenology.

To draw a regular polygon in LOGO, children could program a "turtle" to move forward an x number of steps then turn y degrees, repeating this action sequence a total of z times. If a child were to repeat the sequence of walking forward 10 feet and turning right 90 degrees four times in the physical world, s/he would trace the path of a square with a perimeter of 40 feet. If the turtle were programmed to draw its path, a child would see a square being drawn, and perhaps learn about iterative function. According to Papert, creating a square using LOGO provides learners with an alternative cognitive structuration to traditional school-based interpretations of Euclidean geometry [1]. In this respect, what a square *is* is contingent on the perceptuomotor schema mediating its apprehension and/or construction: a square can be a shape defined by four right angles and four equal sides; yet it might just

IDC 2012, June 12–15, 2012, Bremen, Germany.

as well be the product of a vicarious, situated re-enactment of the child's lived experience.

Papert's vision of computer-supported learning was originally formulated in an era in which the primary user interface for computers was the keyboard, and later the mouse [10, 20]. In the subsequent decades following the publication of *Mindstorms*, a new paradigm of human computer interaction that combines physical interactions with tangible artifacts has emerged [14]. Today, the notion of inputting a user's actual physical activity directly into a computational environment is no longer a dream, but a rapidly evolving reality.

2.2 Identifying the Links Between Interaction and Cognition

While commercial wireless input technologies now allow for increasingly novel and interesting ways for users to physically interact with a computational program (i.e., Nintendo Wii, Xbox Kinect, and smartphones), we must always bear in mind that an *instructional* activity should provide meaningful opportunities for learning.

One could argue that the utility of any instructional technology is ultimately contingent on the manner and degree to which it supports learners' cognition. Therefore, a potentially critical question for instructional designers is how to synthesize theories of cognition and learning with the affordances of tangible and ubiquitous computing devices.

Towards this goal, we propose that the design process should take into consideration the manner in which learners' *intra*-subjective multi-modal perceptions and prior funds of knowledge interact within a designed, instructional situation. An objective of this work is to present a general model of interaction and cognition to guide the instructional design process. The proposed model is framed in large part by the distinction between schemes and situations made by Gerard Vergnaud [28].

2.3 Evoking Schemes and Enacting Situations

Piaget originally adapted Kant's notion of "scheme" to describe the goal-oriented, sequential organization of an individual's sensory-motor activity in the world [23]. Elaborating on Piaget, Vergnaud highlights the relationship between an individual's (e.g., mathematical) schemes on the one hand, and the particular situations that give rise to and transform said schemes.

In brief, the schemes an individual already possesses determine the forms and organization of activity the individual applies towards novel situations. Reciprocally, the situations an individual encounters determine the schemes that are elicited and/or constructed.

Because Vergnaud never offers a formal definition of what he means by situation, we will loosely describe situations in terms of Schank's goal-based scenarios [26]. In this sense, an *instructional* situation is comprised of goal(s) for learners to accomplish, as well as specific means and mechanisms for accomplishing them (what Schank refers to as 'scripts').

Consider the case of teaching someone to play chess. In order to teach them the schemes of movement and play for a given piece, say the bishop, one must first establish an instructional goal (e.g. "learning how pieces move"), and present them with situations in which the movement of said piece could be demonstrated. Alternately, if someone already knew how to play chess, encountering a situation involving chess pieces, could lead them to think of particular schemes of movement/play.

This subjective coupling of situations and schemes occurs as an *accommodation* of existing schemes. In other words, as one encounters new situations, the schemes one may apply to a given class of situations can change. A person may also *assimilate* aspects of a novel situation into an existing scheme if it is amenable for the application of the scheme per some task demand. Consider now the chess notion of "forking," in which a player's piece is positioned to threaten two different opponent pieces at once. Encounters with situations in which this stratagem can or have been applied will arguably change how one learns to utilize one's pieces.

Disambiguating this relationship between scheme and situation may prove useful to instructional designers. After all, the objective of an instructional design is to support learners' development of specific concepts/schemes. An instructional designer begins with a concept/scheme in mind, and attempts to engineer a representative situation. The designer's ultimate intention is to steer learners along a particular cognitive trajectory by choreographing an instructional situation for them to experience. This may involve instantiating specific rules and causal relationships between input and perceptual output for the learner to experience. Consequentially, an objective of our work is to delineate and define mechanisms for describing how the situations learners' experiences contribute towards the emergence of new concepts/schemes.

2.3.1 Evocation

It is universally accepted that familiar artifacts, symbols, and signs can activate an individual's pre-existing schemes. Tasking students with a particular goal may also lead them to draw upon pre-existing problem solving strategies/ heuristics. Accordingly, we use the verb "to evoke" to refer to the activation of students' prior schemes upon encountering an instructional situation.

Similarly, students' schemes may be evoked upon their discovery of familiar features present in an instructional situation. By anticipating the schemes that are likely to be evoked by particular design features, designers could be better prepared to interpret and address students' responses as they guide them towards the desired learning objective.

To quickly illustrate, consider the image below (Figure 1). In the event that you are familiar with the game of chess, this image may evoke particular concepts/schemes related to chess.

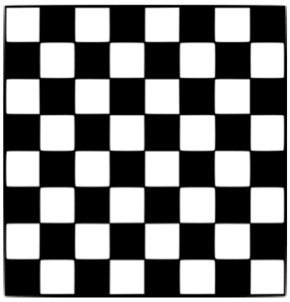

Figure 1: This image might evoke schemes related to chess if and only if one has previously learned chess.

Of course, if you never learned about chess, the image below would simply be an arrangement of 64 squares, or perhaps, a checkerboard (if you knew checkers), or grandmother's kitchen tiles (if her tiles had that pattern). Note that even if a situation appears decidedly unfamiliar, *some* aspect of one's prior-knowledge seems to be evoked.

2.3.2 Enactment

According to Piaget, "in order to know objects, the subject must act upon them: he must displace, connect, combine, take apart, and reassemble them" [23, p. 704]. This constructivist epistemological position necessarily implies that a designer cannot directly influence the development of learners' cognitive schemes—the designer can at best *in*directly guide this development by deliberately orchestrating interactions between evoked prior schemes and situated perceptuomotor operations. We posit that a key objective of interaction design for learning is to guide learners to create, alter, and/or attend to situated phenomena that can be assimilated into and/or lead to the accommodation of the learners' existing cognitive schemes. Furthermore, we observe that an instructional designer may choose to deliberately engender discontinuities in learners' existing schemes—a state Piaget referred to as disequilibrium. This may occur when the learner's experience of the enacted situation is at odds with their pre-existing schemes and/or expectations.

Therefore, in tandem with "evoke" we use "enact" to describe learners' purposeful organization of their dynamic, in-the-moment interactions with and perceptions of an instructional situation. To elaborate, designers specify the rules and causal relationships that govern learners' actions, establish the goals of an instructional activity, and pre-determine the specific modes of user input and resulting forms of perceptual output. As such, designers plan the actions/transformations of a dynamically unfolding situation *to be* enacted by a learner.

The learner, on the other hand, encounters, executes and experiences the designed situation. We define this "enactment," as noun, as the sum of actions and outcomes that are experienced as learners attempt to complete some task objective. Bruner [5] originally used the term "enactive representation" to elucidate how children's actions and activities in the world contribute to their development of iconic and/or symbolic forms of representation. Bruner had argued that only after "something" is first acted upon and experienced in the world can it be referenced, first as an object of thought and/or later as an icon and/or symbol. A potential implication of Bruner's framework is to conceptualize learning in terms of the coordination between grounded experience and symbolic abstraction ([24], chapter 8). Here, we concur with Noss and Hoyles [19] in viewing "abstraction" as arising from grounded experience.

Relating these ideas to recent work on 'embodied cognition' [2,14,27], we believe that when learners are guided to generate, alter, and/or attend to specific phenomena, they are fostering multi-modal image schemas, the experience of which can be assimilated into, and lead to the accommodation of their cognitive schemes.

To help illustrate, consider the case of chess again. How might someone learn how different chess pieces move? Arguably, in addition to written and/or verbal explanations, the most effective way for someone to learn how each piece moves is to actually move (or observe someone moving) said pieces by their respective rules. Similarly, to communicate the concept of "forking," the learner must experience some situation wherein this concept is demonstrated. Whether or not the pieces are physically or digitally instantiated appears to be of secondary concern—the main point is that the scheme(s) to be learned must first be reproduced/simulated as perceivable actions in the world. This demonstrated and/or experienced enactment of the instructional situation (by teacher and/or student) provides what Papert might have described as the proverbial "object(s) to think with." To paraphrase Piaget, the enacted situation functions as the experiential basis for learners' reflective abstraction [23].

2.4 A 3-Parameter Model of an Instructional System.

Taken together, the constructs of evocation and enactment are intended to help delineate and describe the continuous process of conceptual development that can occur as students encounter, enact, and in so doing attempt to make sense of instructional situations.

To locate these constructs within a broader heuristic design framework, we now propose the following *3-Parameter Model of an Instructional System* (Figure 2, below), in which learners' emergent scheme(s) are conceptualized as the aggregate product(s) of interactions among: (1) prior knowledge schemes; (2) the instructional situation, which describe here as encompassing the goals media, semiotic artifacts, and embedded input–output functional contingencies; and (3) learners' actions and perceptions.

Here we call the reader's attention to the interdependent nature of each of the three parameters. The reader will observe Vergnaud's theoretical pairing between scheme and situation as indicated by the bi-directional relationship between prior schemes and the instructional situation (see Figure 2). The model also suggests that the actions and perceptions available to an individual are a function of both prior experience [12] and the designed affordances of the situation itself [11].

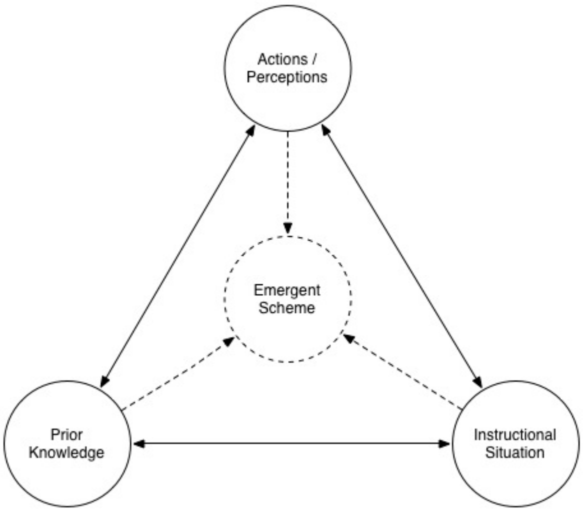

Figure 2: 3-Parameter Model of an Instructional System in which learners' prior knowledge, the instructional activity, and perceptual experience contribute towards an emergent scheme.

One important caveat we wish to note is that while there doubtless exists a qualitative difference between vicariously as opposed to directly performing actions, it appears that contingent upon the learner's goal orientation, a cognitive interaction between prior schemes and the instructional situation occurs in either circumstance [27].

3. APPLYING THE 3-PARAMETER MODEL

To support the plausibility of the proposed model, we now demonstrate how it elucidates findings from the implementation of two instructional designs in the literature (Example 1 & 2, below). Following that, we will describe how the model has informed the development of our own design, the *Bar Graph Bouncer* (Ex. 3).

3.1 Example 1: LOGO Programming

Revisiting the case of drawing geometric objects in LOGO, we observe that the activity depends on evoking students' pre-existing knowledge about polygons. While students are eventually encouraged to experiment and draw whatever they wish, instruction initially begins with an instructor's defined goal such as attempting to draw familiar shapes. Students are then expected to use the programming syntax to accomplish this task.

Framing LOGO programming in terms of the proposed 3-parameter model, we identify the following factors as essential to the emergence of a concept such as a "turtle geometry square": 1.) Evoking a learner's embodied and conceptual knowledge of walking, turning, distance, etc. (prior knowledge); 2.) The instructional situation comprised of goals/objectives that the instructor establishes (making a square), the designed activity (iterative steps and turns), the LOGO syntax, and the computational display output; and 3.) Dynamically enacting the movement of a physical (i.e., robotic) or digital turtle 'walking forward' on a display (actions and perceptions).

Here, we interpret the computational instantiation of a shape that learners perceive the turtle drawing as a visual enactment of the concept(s) to be learned. Invariably, programming errors often occur, and as a result, unexpected drawings are often made. Thus, the concepts students might eventually learn through programming with LOGO such as squares, recursion, loops, etc., emerge as a result of an interaction between prior knowledge, the goals and media of the instructional situation, and learner's perception of the turtle's expected and unexpected actions.

3.2 Example 2: Mathematical Image Trainer

The Mathematical Image Trainer developed by the Embodied Design Research Laboratory utilized the infrared sensor on a Nintendo Wii controller so as to enable users to develop perceptuomotor schemes viewed as supporting the development of proportional transformation [13]. The project was driven by the conjecture that some mathematical concepts are difficult to learn because life does not occasion naturalistic opportunities to interact in ways that foster the development of the multimodal images schemas assumed to underlie the target concept.

Students begin the activity with a red screen. In order to make the screen green, students must learn to position the pointers at proportional distances from the baseline, such as 1:2 ratio (e.g., at 1"& 2", 3" & 6", 10" & 20", etc., above the baseline). Whereas most students discovered how to make the screen green on their own, students could not describe their actions in formal mathematical terms until a Cartesian coordinate grid and numerals were introduced.

The instructional situation was designed to provide students with physically embodied bases for understanding concepts such as ratio and proportion. Another explicit consideration of the design was to account for and challenge students' tendencies to apply additive rules of reasoning when presented with multiplicative relationships such as ratio.

The system was designed so that learners could dynamically enact two different classes of situations: the generation of a red or green screen. The green screen would only appear if the distances between the student's left and right hands and a table were at a given proportion (e.g., a 1 to 2 ratio). Otherwise, the screen would be red when the hand positions where not at the set ratio.

Figure 3: Infrared sensors transmit the relative locations of the student's hands to a Wii remote. The screen turns green if the hands are held at a proportional distance.

The Mathematical Image Trainer can be retrospectively analyzed in terms of the proposed 3-parameter model as follows: 1.) The design relied on evoking students socio-normative associations of color (i.e. green = go/good, red = stop/bad) as well as their fluency with grids and numerical coordinates. 2.) The instructional situation was designed such that students would attend to the relative distances between their hands and a surface. The learner's goal orientation in the activity shifts from simply "making the screen green," to discovering the mathematical relationship governing the interaction, and 3.) The screen color that students immediately perceived (red/green) corresponded to students' relative hand positions as they moved their hands up and down.

3.3 Example 3: The "Bar Graph Bouncer"

Our current work-in-progress, entitled the "Bar Graph Bouncer" is intended to support young children's conceptualization of number, develop their ability to interpret graphic representations of data such as bar and line graphs, and encourage physical exercise (Figure 4).

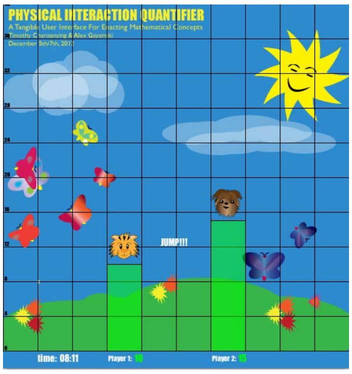

Figure 4: A dynamic bar graph visualization

3.3.1 Body Syntonicity, Redux

In keeping with Papert's theme of relating children's bodily understanding about their own actions with mathematical notions and forms of representation, we designed the Bar Graph Bouncer activity so that children could observe a direct correlation between their activity and the real-time transformation of a bar graph (see also [18]).

3.3.2 Applying the 3-Parameter Model

We identified the following factors as potentially vital to helping young children learn to interpret graphic representational tools such as bar and line graphs: 1.) Evoking a learner's understanding of the one-to-one correspondence between repeated jumping and number (prior knowledge); 2.) The goals/objectives of the instructional activity, in this case, to jump more times than their peers (the instructional situation); and 3.) The synchronicity and dynamic enactment of each jump and of the visualization growing right before their very eyes (actions and perceptions).

To elaborate, the interactive graphic visualization grows each time a user physically jumps up (Figure 5). The more times the user jumps, the taller the graph becomes. The visualization screen also shows the jump count of each user numerically at the bottom. While this observation may seem fairly obvious to a knowledgeable adult, learning how to interpret a graphic representation is a non-trivial task for a young child [16]. The juxtaposition of students' activity (jumping up and down side by side with another player) and the dynamically changing visualization that reflect their activity can help make this more concrete for children.

Figure 5: The visualizations keeps track of each time a user physically jumps up and down in real-time.

3.3.3 Technical Implementation

The current iteration of the Bar Graph Bouncer uses data from a smart phone accelerometer to calculate the number of jumps an individual takes. An Android application was written to calculate vertical acceleration. This data is then transmitted wirelessly over a WiFi-enabled local-area network to a nearby computer as a UDP packet. The open source programming language, *Processing,* was used to translate the UDP packet into visual output.

3.4 Related Work

3.4.1 Graphing Students' Poistions

Nemirovsky, Tierney, and Write [18] have previously used motion detection technology to calculate and visually graph students' relative distances from a stationary sensor. The *SMALLAB* learning environment [4] combines visualization and location sensing to technology to create embodied and interactive learning environments of a classroom size scale.

3.4.2 Combining Physical Activity and Games

The Nintendo Wii greatly popularized and commercialized the combination of physical action and video games [22]. The Wii remote interface allowed developers to design interactions that roughly translated users' physical actions into electronic gestures. In more recent years, Xbox Kinect using the depth sensing technology has supported a more direct mapping between bodily action and game play control without any physical controllers.

3.4.3 Augmented Reality, Ubiquitous Gaming & Exercise

Human Pacman combines augmented vision, mobile computing, wireless LAN, and motion-tracking technologies to transform the physical world into a digitally enhanced play space [8]. The designers of *Touch & Learn* have used near-field communication (NFC) tags both to help students learn to read and encourage them to run around outdoors [25].

Games are also being designed to encourage people to exert invest significant physical effort as part of the gameplay and to facilitate interaction [9, 17, 29].

4. FUTURE WORK

The Bar Graph Bouncer is presented here as working proof-of-concept. However, we are currently implementing and evaluating the utility of this design in elementary school classrooms.

Following the ethos of design-based research [5], we plan to continue further refining our theoretical frameworks as well as develop new tools for instruction.

From a technical standpoint, we envision being able deploy inexpensive, real time location-sensitive systems in venues such as classrooms, gymnasiums and school-yards that can accurately track the movements and positions of students as they engage in various forms of physical activity, such as relay races or playing a game such as dodge-ball. We could then selectively visualize quantitative properties such as the speed of students (e.g., number of yards/per second they can run) or the fraction/percentage of players remaining in a dodge-ball contest.

5. CONCLUSIONS

For Papert, the mathematical understandings that students might construct from body-syntonic activities were seen as powerful cognitive entries into normative mathematical practice. He believed that mathematics could be more meaningful and accessible to students if it were intimately connected to their experience of the world around them. Now that technologies for realizing Papert's forward thinking vision of education have become increasingly widespread and affordable, we believe the time has come to re-theorize our principles of instructional design as well.

Building on developmental and embodiment theories of cognition, we have proposed two mechanisms for characterizing learners' interactions with an instructional artifact or activity. First, we highlight the fact that instructional activities can evoke pre-existing schemes in the mind of the learner. Second, we use the term "enact" to describe how learners' dynamic, situated activity can be structured so as to help facilitate their construction of a given scheme. To unify these insights into a coherent framework for analyzing instructional design, we have proposed a 3-Parameter model that positions designers to better account for the otherwise unexpected ways that a student might make sense of a novel instructional situation.

Conceptualizing instructional practice in terms of coordinating between the evocation of prior-knowledge and the construction of new schemes vis-à-vis the enactment of specific situations may provide designers and researchers alike with a useful shared vernacular that bridges the worlds of cognition, learning, and design. More importantly perhaps, this approach to framing instructional interactions may lend useful insights for anticipating and evaluating the effectiveness of instructional designs.

Lastly, in attempting to couple vigorous physical action with mathematical learning, we wish to lend support to broader efforts to promote physical fitness among children.

6. ACKNOWLEDGMENTS

We wish to thank Dor Abrahamson of the Embodied Design Research Laboratory, UC Berkeley Graduate School of Education, and Deepak Subramanian and our colleagues at the UC Berkeley School of Information. The research reported here was supported in part by the Institute of Education Sciences pre-doctoral training grant R305B090026 to the University of California, Berkeley. The opinions expressed are those of the authors and do not represent views of the Institute of Education Sciences or the U.S. Department of Education

7. REFERENCES

[1] Abelson H., & diSessa A. (1981) *Turtle geometry.* MIT Press, Cambridge, Massachusetts

[2] Abrahamson, D., & Trninic, D. (2011). Toward an embodied-interaction design framework for mathematical concepts. In P. Blikstein & P. Marshall (Eds.),*Proceedings of IDC 2011* (Vol. Full Papers, pp. 1-10), Ann Arbor, MI, June 20-23. IDC

[3] Barsalou, L. W. (2010). Grounded cognition. *Topics in Cognitive Science, 2*, 716-724.

[4] Birchfield, D., Ciufo, T., & Minyard, G. (2006). SMALLab: a mediated platform for education. ACM SIGGRAPH 2006 Educators program (p. 33). Boston, Massachusetts: ACM.

[5] Brown, A. L. (1992). Design experiments. *Journal of the Learning Sciences, 2*(2), 141-178.

[6] Bruner, J. (1966). Towards a theory of instruction. Cambridge, MA: Harvard University Press.

[7] Charoenying, T. (2011). The evocation and enactment of conceptual schemes. In T. Lamberg (Ed.), *Proceedings of the 33rd Annual Meeting of the North-American Chapter of the International Group for the Psychology of Mathematics Education* (Vol. 1, pp. 156-164). Reno, NV

[8] Cheok, A. D., Goh, K. H., Liu, W., Farbiz, F., Fong, S. W., Teo, S. L., Li, Y., et al. (2004). Human Pacman. *Personal Ubiquitous Computing, 8*(2), 71-81.

[9] Consolvo, S., Everitt, K., Smith, I., & Landay, J. A. (2006). Design requirements for technologies that encourage physical activity. *Proceedings of the SIGCHI conference on Human Factors in computing systems* (pp. 457-466). Montreal, Quebec, Canada: ACM.

[10]Dourish P (2001) *Where the action Is*. MIT Press, Cambridge, MA

[11]Gibson, J.J. (1977), The Theory of Affordances. In *Perceiving, Acting, and Knowing*, Eds. Robert Shaw and John Bransford, ISBN 0-470-99014-7.

[12]Goodwin, C. (1994). Professional Vision. *American Anthropologist, New Series, 96*(3), 606-633.

[13]Howison, M., Trninic, D., Reinholz, D., & Abrahamson, D. (2011). The Mathematical Imagery Trainer. In G. Fitzpatrick, C. Gutwin, B. Begole, W. A. Kellogg, & D. Tan (Eds.), *Proceedings of CHI 2011*, Vancouver, May 7-12, 2011 (Vol. pp. 1989-1998). ACM: CHI.

[14]Ishii H, Ullmer B (1997) Tangible bits: toward seamless interfaces between people, bits and atoms. In: *Proceedings of CHI, 1997*. ACM Press, New York, pp 234–241.

[15]Lakoff, G., & Nunez, R. (2000). *Where mathematics comes from*. Basic Books.

[16]Leinhardt, G., Zaslavsky, O., & Stein, M. M. (1990). Functions, graphs, and graphing: Tasks, learning and teaching. *Review of Educational Research*, 60, 1-64.

[17]Mueller, F. "Floyd," Gibbs, M. R., & Vetere, F. (2009). Design influence on social play in distributed exertion games. *Proceedings of the 27th international conference on Human factors in computing systems* (pp. 1539-1548). Boston, MA, USA: ACM.

[18]Nemirovsky, R., Tierney, C., & Wright, T. (1998). Body motion and graphing. *Cognition & Instruction, 16*(2), 119-172.

[19]Noss, R., & Hoyles, C. (1996). *Windows on mathematical meanings: Learning cultures and computers.* Dordrecht: Kluwer.

[20]McNerney, T.S. From turtles to Tangible Programming Bricks: explorations in physical language design, *Personal and Ubiquitous Computing*, v.8 n.5, p.326-337, September 2004

[21]Papert, S. (1980). *Mindstorms: Children, computers, and powerful ideas*. New York, NY: Basic Books.

[22]Parker, J.R. Games for Physical Activity: A Preliminary Examination of the Nintendo Wii, *6th International Symposium on Computer Science in Sport*, Calgary. June 3- 6, 2007.

[23]Piaget, J. (1983). Piaget's theory. In P. Mussen (ed.). *Handbook of Child Psychology.* 4th edition. Vol. 1. New York: Wiley.

[24]Raffle, H. *Sculpting Behavior.* Ph.D. Thesis. Massachusetts Institute of Technology (2008).

[25]Sanchez, I., Cortes, M., Riekki, J., & Oja, M. (2011). NFC-based interactive learning environments for children. *Proceedings of the 10th International Conference on Interaction Design and Children* (pp. 205-208). Ann Arbor, Michigan: ACM.

[26]Schank, R. (1996). Goal-based scenarios: Case-based reasoning meets learning by doing. In D.Leake (Ed.), Case-Based Reasoning (pp. 295-348). Cambridge, MA: MIT Press.

[27]Sperry, R.W. (1952). Neurology and the mind-body problem. *American Scientist, 40*, 291-312.

[28]Vergnaud, G. (2009). The theory of conceptual fields. In T. Nunes (Ed.), Giving meaning to mathematical signs:. *Human Development [Special Issue] 52*(2), 83 – 94.

[29]Yim, J., & Graham, T. C. N. (2007). Using games to increase exercise motivation. *Proceedings of the 2007 conference on Future Play* (pp. 166-173).Toronto: ACM.