Ads and Attention

Hal Varian

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Google™
Outline

▶ Online ad auctions
▶ Newspaper ads
▶ TV ads
▶ Stock purchases
▶ Commodity ads
Search results and ads
Why ad auction?

Why sell ads via an auction?

- Millions of keywords to price
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- Positions (slots) have quite different values
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- Advertisers want to change bids in real time
- Cost of entry to auction by advertisers is very low
- Auction does a very good job at rent extraction
Notation

- $a = \text{index for advertiser } a = 1, \ldots, A$
- $s = \text{index for slots } s = 1, \ldots, S$
- $b_a = \text{bid per click of advertiser } a$
- $v_a = \text{value per click of advertiser } a$
- $p_s = \text{equilibrium price per click paid by the advertiser in slot } s$
- $e_a = \text{advertiser-specific CTR for advertiser } a$
- $x_s = \text{position-specific CTR for slot } s. \text{ Assume } x_s > x_{s+1}$.
- $z_{as} = \text{clickthrough rate of advertiser } a \text{ in slot } s$
  - actual clicks = ad CTR $\times$ position CTR
  - $z_{as} = e_a \times x_s$
Rules of auction

Rules of Generalized Second Price auction (GSP)

1. Each advertiser \(a\) chooses a bid \(b_a\).
2. The advertisers are ordered by bid times adv-specific CTR rate \((b_a e_a)\).
3. The price that advertiser \(a\) pays for a click is the minimum necessary to retain its position.
4. If there are fewer bidders than slots, some slots are unfilled and the last bidder pays a reserve price \(r\).

Ranking:

\[ b_1 e_1 > b_2 e_2 > \ldots > b_m e_m \]

Pricing:

\[
\begin{align*}
 p_s e_s &= b_{s+1} e_{s+1} \\
 p_s &= \frac{b_{s+1} e_{s+1}}{e_s}
\end{align*}
\]
Rationale

▶ **Question.** Why not just rank ads by bid for clicks?

Better to rank ads by expected revenue. Price times quantity is what matters. Joe's Jets has high-value click but few sales. Moe's Models has low-value click but many sales. Auction is really an auction for impressions. bid per impression = bid per click × clicks per impression. search engine has impressions to sell, but advertisers want to pay for clicks. I want to pay in dollars, you want to get paid in euros. Clickthrough rates is the exchange rate. For rest of this exposition, assume e ≡ 1.

Hal Varian Ad-tention
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▶ **For rest of this exposition, assume** $e_a \equiv 1$. 
Equilibrium conditions

Nash condition: adv in position $s + 1$ doesn’t want to move to up to $s$

\[
(v_{s+1} - p_{s+1})x_{s+1} \geq (v_{s+1} - p_s)x_s
\]

\[
p_s x_s \geq p_{s+1} x_{s+1} + v_{s+1}(x_s - x_{s+1})
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Interpretation

- advertisers compete for *incremental* clicks ($x_s - x_{s+1}$)
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Interpretation

- advertisers compete for *incremental* clicks $(x_s - x_{s+1})$
- amount adv in slot $s$ has to pay is enough to beat adv in $s + 1$
- auction is efficient in that it awards best positions to adv with highest values
Equilibrium revenue

Start with Nash condition:

\[ p_s x_s \geq p_{s+1} x_{s+1} + v_{s+1} (x_s - x_{s+1}) \]

Solve the recursion

\[
\begin{align*}
p_1 x_1 &\geq v_2 (x_1 - x_2) + v_3 (x_2 - x_3) + v_4 (x_3 - x_4) + \cdots + p_m x_m \\
p_2 x_2 &\geq v_3 (x_2 - x_3) + v_4 (x_3 - x_4) + \cdots + p_m x_m \\
p_3 x_3 &\geq v_4 (x_3 - x_4) + \cdots + p_m x_m \\
\end{align*}
\]

Add up to get

\[
\sum_{s} p_s x_s \geq v_2 (x_1 - x_2) + 2v_3 (x_2 - x_3) + 3v_4 (x_3 - x_4) + \cdots + (m-1)p_m x_m.
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Revenue bounds

Also true in equilibrium that advertiser doesn’t want to move down. Same sort of calculations give us an upper bound. (See paper for qualifications.)

\[
R_{dn} = v_2(x_1 - x_2) + 2v_3(x_2 - x_3) + \ldots + (m - 1)p_m x_m.
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\[
R_{up} = v_1(x_1 - x_2) + 2v_2(x_2 - x_3) + \ldots + (m - 1)p_m x_m.
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- Equilibrium revenue is between the two bounds
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- But here get explicit solution
What if attention was free?

- Suppose user was equally likely to click on each ad

Revenue reduces to $R_{dn} = (m - 1) p^m x_m$.

This is just an $m$th price auction. The difference between the two is the value of attention. Publisher gets to capture that value (search, TV, print).
What if attention was free?

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- In that case, all clickthrough rates are equal: $x_1 = x_2 \cdots x_m$. 

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Much concern about economics of newspapers
Newspapers

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- One claim: ad rates are much higher for offline
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- One claim: ad rates are much higher for offline
- But is that really true?
Online and offline attention

- Offline news
  - Spend 24 minutes a day reading
  - Generates 88% of ad revenue
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- If readers spent the same amount of time in online reading, problems would go away
Online ad auctions
Newspapers
TV
Stocks
Food

Why the difference?

- Offline reading is leisure time activity
- Online reading is labor time activity
TV and YouTube

- **TV**
  - CPM is about $10
  - 20 ads in 1 hour
  - TV advertisers pay 20 cents/hour for your attention

- **YouTube**
  - CPM is about $1
  - 1 ad impression in 4 minutes
  - YouTube advertisers pay 15 cents/hour for your attention

- When a stock is “in the news” individual investors are much more likely to buy
- True for good news or bad news
- Asymmetry between buying and selling: individuals can only sell a few stocks but can buy many
Food ads

- Agricultural marketing boards
  - “Got milk?”
  - “It’s the cheese.”
  - “Pork, nature’s other white meat.”
  - “The incredible, edible egg.”
- But they work!
  - $1 billion per year in California alone
  - $1 spent on ads generates $3 to $6 of incremental revenue
Dancing raisins

- California raisins increased raisin sales 10 percent
- California raisin board: before the campaign raisens were “at best dull and boring” but afterwards, people were no longer “ashamed to each raisins.”
- “. . .otherwise people might forget to eat raisins.”
Raisin collectibles