Public Goods and Private Gifts

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Tying public and private goods

- Public TV
- Charity ball, parties for donors
- Coase and lighthouses
- Kickstarter
  - Web Fonts
  - YouTube channel
- Micropatronage, crowd funding
Example of a Kickstarter project

An Illustrated Guide to Income in the United States
by Catherine Mulbrandon

Funded! This project successfully raised its funding goal on May 1, 2011.

A comprehensive collection of infographics, maps and charts looking at the history of incomes and occupations in the U.S.
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I want to create a collection of clear, easy-to-understand infographics about incomes and occupations that will help people understand the history of income in the United States.

THE BACKGROUND

Seven years ago, as part of my master’s thesis, I created a poster “How Much Do You Earn?” in order to visualize the income distribution in the United States. However, I discovered that income was more complex and interesting than I originally thought. So I continued my research and began posting new infographics to a website: VisualizingEconomics.com. While I cover other subjects that catch my fancy (stock markets, growth of countries, taxes), I keep returning to income as I find new and interesting data. You may have seen my work at Slate.com, NPR.org, WashingtonPost.com, MSNBC, PBS Newshour and on popular financial blogs. VisualizingEconomics.com is recommended as a resource for teachers by the Library of Congress. Also some graphs have been reproduced in college textbooks.

The Average Income graph below is one of the most popular posts on my site. Other examples of my work can be found at VisualizingEconomics.com

THE PROJECT

What is the income for different types of jobs?... Who makes the most money?... How does income in United States compare to other countries... How has income changed since the 1920s? 1950s? 1980s?
Notation

- $g_i =$ contribution by person $i$
- $G = \sum_i g_i =$ total contributions
- $U_i(x_i, G, g_i) =$ utility depends on private consumption, public good, donor’s contribution (which determines private gift according to size)
- Same as Andreoni’s “warm glow”
- But in many cases gifts are discrete: there is a threshold for the “glow”
Discrete model is of interest due to Bagnoli-Lipman and provision point mechanisms.

Contributions summing to threshold can be a Nash equilibrium.

Obviously, equilibrium not unique.

However, it is not a Nash equilibrium for sums to exceed threshold.

What about Kickstarter?
Hot or not model with 2 agents

\[ u_i = \text{value of public good to agent } i \]
\[ r_i = \text{value of private good to agent } i \]
\[ g_i = \text{contribution of agent } i \]
\[ G = g_1 + g_2 = \text{total contributions} \]
\[ \bar{G} = \text{threshold for total contributions to public good} \]
\[ \bar{g}_i = \text{threshold for contribution to receive gift} \]
Payoff to agent 2

\[ v_2 = \begin{cases} 
  u_2 + r_2 - g_2 & \text{if } g_1 + g_2 \geq \bar{G} \text{ and } g_2 \geq \bar{g}_2 \\
  u_2 - g_2 & \text{if } g_1 + g_2 \geq \bar{G} \text{ and } g_2 < \bar{g}_2 \\
  0 & \text{if } g_1 + g_2 < \bar{G} 
\end{cases} \]
Direct utility of agent 2 is

$$\max\{u_2 - g_2, 0\} \tag{1}$$

Since \(g_1 + g_2 = \bar{G}\), indirect utility of agent 2 is

$$v_2(g_1) = \max\{u_2 - \bar{G} + g_1, 0\} \tag{2}$$

Agent 2 is indifferent between contributing or not at

$$\hat{g}_1 = \bar{G} - u_2.$$ 

Reaction function for agent 2:

$$g_2(g_1) = \begin{cases} \bar{G} - g_1 & \text{if } g_1 \geq \hat{g}_1 = \bar{G} - u_2 \\ 0 & \text{otherwise} \end{cases} \tag{3}$$
Classical Nash equilibrium

\[ g_1 \quad g_2 \]

\[ f_1(g_2) \]

\[ f_2(g_1) \]

Nash equilibria

\[ ^\^ G \]

\[ ^\^ \]

\[ ^\^ g_1 \quad g_2 \]

\[ ^\^ \]

\[ ^\^ G \]

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Hot or not model

\[ v_2(g_1) = \begin{cases} 
  u_2 + r_2 - g_2 & \text{if } g_1 + g_2 \geq \bar{G} \text{ and } g_2 \geq \bar{g}_2 \\
  u_2 - g_2 & \text{if } g_1 + g_2 \geq \bar{G} \text{ and } g_2 < \bar{g}_2 \\
  0 & \text{if } g_1 + g_2 < \bar{G} 
\end{cases} \] (4)

**Assumption 1.** It is efficient to jointly fund the public good: \( u_1 + u_2 > G \) but \( u_1 < G \) and \( u_2 < G \).

**Assumption 2.** Threshold for private gift is small. \( u_i + r_i > \bar{g}_i \) for \( i = 1, 2 \).

Now agent 2 just begins to contribute when \( \hat{g}_1 = \bar{G} - u_2 - r_2 \), which by assumption is sufficient to trigger the private gift. Region of contribution is larger.
Reaction function for agent 2

g_2(g_1) = \begin{cases} 
0 & \text{if } g_1 < \hat{g}_1 \\
\bar{G} - g_1 & \text{if } \bar{G} - \bar{g}_2 \geq g_1 \geq \hat{g}_1 \\
\bar{g}_2 & \text{if } g_1 > \bar{G} - \bar{g}_2 
\end{cases} \quad (5)
Case 1. Public equilibria

Nash equilibria
Case 2. Partially private equilibrium

\[
\begin{align*}
&g_1 \quad g_2 \\
&\quad f_1(g_2) \\
&\quad f_2(g_1)
\end{align*}
\]

Nash equilibria
Case 3. Fully private equilibrium

\[ f_2(g_1) \]

\[ f_1(g_2) \]

Nash equilibria

\[ g_1 \]

\[ g_2 \]

\[ G \]
Conclusion

- Adding private gift may enlarge the set of equilibria (fully public)
- Adding private gift may contract the set of equilibria (other cases)
- May yield a purely private equilibrium with total contributions greater that $\bar{G}$.
- The more players the larger the contributions due to private gifts
- Contributions to receive private gift may be adequate to fund public good