The Market

A. Example of an economic model — the market for apartments
   1. models are simplifications of reality
   2. for example, assume all apartments are identical
   3. some are close to the university, others are far away
   4. price of outer-ring apartments is exogenous — determined outside the model
   5. price of inner-ring apartments is endogenous — determined within the model

B. Two principles of economics
   1. optimization principle — people choose actions that are in their interest
   2. equilibrium principle — people’s actions must eventually be consistent with each other

C. Constructing the demand curve
   1. line up the people by willingness-to-pay. See Figure 1.1.

2. for large numbers of people, this is essentially a smooth curve as in Figure 1.2.
D. Supply curve
   1. depends on time frame
   2. but we’ll look at the **short run** — when supply of apartments is fixed.

E. Equilibrium
   1. when demand equals supply
   2. price that clears the market

F. Comparative statics
   1. how does equilibrium adjust when economic conditions change?
   2. “comparative” — compare two equilibria
   3. “statics” — only look at equilibria, not at adjustment
   4. example — increase in supply lowers price; see Figure 1.5.
   5. example — create condos which are purchased by renters; no effect on price; see Figure 1.6.

G. Other ways to allocate apartments
   1. discriminating monopolist
   2. ordinary monopolist
   3. rent control
H. Comparing different institutions
1. need a criterion to compare how efficient these different allocation methods are.

2. an allocation is **Pareto efficient** if there is no way to make some group of people better off without making someone else worse off.

3. if something is *not* Pareto efficient, then there *is* some way to make some people better off without making someone else worse off.

4. if something is not Pareto efficient, then there is some kind of “waste” in the system.

I. Checking efficiency of different methods
   1. free market — efficient
   2. discriminating monopolist — efficient
   3. ordinary monopolist — not efficient
   4. rent control — not efficient

J. Equilibrium in long run
   1. supply will change
   2. can examine efficiency in this context as well
Budget Constraint

A. Consumer theory: consumers choose the best bundles of goods they can afford.
   1. this is virtually the entire theory in a nutshell
   2. but this theory has many surprising consequences

B. Two parts to theory
   1. “can afford” — budget constraint
   2. “best” — according to consumers’ preferences

C. What do we want to do with the theory?
   1. test it — see if it is adequate to describe consumer behavior
   2. predict how behavior changes as economic environment changes
   3. use observed behavior to estimate underlying values
      a) cost-benefit analysis
      b) predicting impact of some policy

D. Consumption bundle
   1. \((x_1, x_2)\) — how much of each good is consumed
   2. \((p_1, p_2)\) — prices of the two goods
   3. \(m\) — money the consumer has to spend
   4. budget constraint: \(p_1 x_1 + p_2 x_2 \leq m\)
   5. all \((x_1, x_2)\) that satisfy this constraint make up the budget set of the consumer. See Figure 2.1.

---

![Figure 2.1](image-url)
E. Two goods
1. theory works with more than two goods, but can’t draw pictures.
2. often think of good 2 (say) as a composite good, representing money to spend on other goods.
3. budget constraint becomes \( p_1 x_1 + x_2 \leq m \).
4. money spent on good 1 \((p_1 x_1)\) plus the money spent on good 2 \((x_2)\) has to be less than or equal to the amount available \((m)\).

F. Budget line
1. \( p_1 x_1 + p_2 x_2 = m \)
2. also written as \( x_2 = m/p_2 - (p_1/p_2)x_1 \).
3. budget line has slope of \(-p_1/p_2\) and vertical intercept of \(m/p_2\).
4. set \( x_1 = 0 \) to find vertical intercept \((m/p_2)\); set \( x_2 = 0 \) to find horizontal intercept \((m/p_1)\).
5. slope of budget line measures opportunity cost of good 1 — how much of good 2 you must give up in order to consume more of good 1.

G. Changes in budget line
1. increasing \( m \) makes parallel shift out. See Figure 2.2.
2. increasing $p_1$ makes budget line steeper. See Figure 2.3.
3. increasing $p_2$ makes budget line flatter
4. just see how intercepts change
5. multiplying all prices by $t$ is just like dividing income by $t$
6. multiplying all prices and income by $t$ doesn’t change budget line
   a) “a perfectly balanced inflation doesn’t change consumption possibilities”

H. The numeraire
1. can arbitrarily assign one price a value of 1 and measure other price relative to that
2. useful when measuring relative prices; e.g., English pounds per dollar, 1987 dollars versus 1974 dollars, etc.

I. Taxes, subsidies, and rationing
1. quantity tax — tax levied on units bought: $p_1 + t$
2. value tax — tax levied on dollars spent: $p_1 + \tau p_1$. Also known as *ad valorem* tax
3. subsidies — opposite of a tax
   a) $p_1 - s$
   b) $(1 - \sigma)p_1$
4. lump sum tax or subsidy — amount of tax or subsidy is independent of the consumer’s choices. Also called a head tax or a poll tax

5. rationing — can’t consume more than a certain amount of some good

J. Example — food stamps
1. before 1979 was an *ad valorem* subsidy on food
   a) paid a certain amount of money to get food stamps which were worth more than they cost
   b) some rationing component — could only buy a maximum amount of food stamps

2. after 1979 got a straight lump-sum grant of food coupons. Not the same as a pure lump-sum grant since could only spend the coupons on food.
Preferences

A. Preferences are relationships between bundles.
   1. if a consumer would choose bundle \((x_1, x_2)\) when \((y_1, y_2)\) is available, then it is natural to say that bundle \((x_1, x_2)\) is preferred to \((y_1, y_2)\) by this consumer.
   2. preferences have to do with the entire bundle of goods, not with individual goods.

B. Notation
   1. \((x_1, x_2) \succ (y_1, y_2)\) means the x-bundle is strictly preferred to the y-bundle
   2. \((x_1, x_2) \sim (y_1, y_2)\) means that the x-bundle is regarded as indifferent to the y-bundle
   3. \((x_1, x_2) \succeq (y_1, y_2)\) means the x-bundle is at least as good as (preferred to or indifferent to) the y-bundle

C. Assumptions about preferences
   1. complete — any two bundles can be compared
   2. reflexive — any bundle is at least as good as itself
   3. transitive — if \(X \succeq Y\) and \(Y \succeq Z\), then \(X \succeq Z\)
      a) transitivity necessary for theory of optimal choice

D. Indifference curves
   1. graph the set of bundles that are indifferent to some bundle.
      See Figure 3.1.
   2. indifference curves are like contour lines on a map
   3. note that indifference curves describing two distinct levels of preference cannot cross. See Figure 3.2.
      a) proof — use transitivity

E. Examples of preferences
   1. perfect substitutes. Figure 3.3.
      a) red pencils and blue pencils; pints and quarts
      b) constant rate of trade-off between the two goods
   2. perfect complements. Figure 3.4.
      a) always consumed together
      b) right shoes and left shoes; coffee and cream
   3. bads. Figure 3.5.
   4. neutrals. Figure 3.6.
   5. satiation or bliss point Figure 3.7.
F. Well-behaved preferences
1. monotonicity — more of either good is better
   a) implies indifference curves have negative slope. Figure 3.9.
2. convexity — averages are preferred to extremes. Figure 3.10.
   a) slope gets flatter as you move further to right
Indifference curves

Satiation point

Figure 3.7

Worse bundles

Better bundles

$(x_1, x_2)$

Figure 3.9

b) example of non-convex preferences
G. Marginal rate of substitution
   1. slope of the indifference curve
   2. \( MRS = \frac{\Delta x_2}{\Delta x_1} \) along an indifference curve. Figure 3.11.
   3. sign problem — natural sign is negative, since indifference curves will generally have negative slope
   4. measures how the consumer is willing to trade off consumption of good 1 for consumption of good 2. Figure 3.12.
   5. measures marginal willingness to pay (give up)
      a) not the same as how much you have to pay
      b) but how much you would be willing to pay
Indifference curve

\[ \Delta x_2 \quad \Delta x_1 \]

Slope = \frac{\Delta x_2}{\Delta x_1} = \text{marginal rate of substitution}

Figure 3.11

Indifference curves

Slope = -E

Figure 3.12
Utility

A. Two ways of viewing utility
   1. old way
      a) measures how “satisfied” you are
         1) not operational
         2) many other problems
   2. new way
      a) summarizes preferences
      b) a utility function assigns a number to each bundle of goods
         so that more preferred bundles get higher numbers
      c) that is, $u(x_1, x_2) > u(y_1, y_2)$ if and only if $(x_1, x_2) \succ (y_1, y_2)$
      d) only the ordering of bundles counts, so this is a theory of ordinal utility
      e) advantages
         1) operational
         2) gives a complete theory of demand

B. Utility functions are not unique
   1. if $u(x_1, x_2)$ is a utility function that represents some preferences, and $f(\cdot)$ is any increasing function, then $f(u(x_1, x_2))$ represents the same preferences
   2. why? Because $u(x_1, x_2) > u(y_1, y_2)$ only if $f(u(x_1, x_2)) > f(u(y_1, y_2))$
   3. so if $u(x_1, x_2)$ is a utility function then any positive monotonic transformation of it is also a utility function that represents the same preferences

C. Constructing a utility function
1. can do it mechanically using the indifference curves. Figure 4.2.

2. can do it using the “meaning” of the preferences

D. Examples

1. utility to indifference curves
   a) easy — just plot all points where the utility is constant

2. indifference curves to utility

3. examples
   a) perfect substitutes — all that matters is total number of pencils, so \( u(x_1, x_2) = x_1 + x_2 \) does the trick
      1) can use any monotonic transformation of this as well, such as \( \log(x_1 + x_2) \)
   b) perfect complements — what matters is the minimum of the left and right shoes you have, so \( u(x_1, x_2) = \min\{x_1, x_2\} \) works
c) quasilinear preferences — indifference curves are vertically parallel. Figure 4.4.
1) utility function has form $u(x_1, x_2) = v(x_1) + x_2$
d) Cobb-Douglas preferences. Figure 4.5.
   1) utility has form \( u(x_1, x_2) = x_1^b x_2^c \)
   2) convenient to take transformation \( f(u) = u^{\frac{1}{b+c}} \) and write \( x_1^{\frac{b}{b+c}} x_2^{\frac{c}{b+c}} \)
   3) or \( x_1^b x_2^{1-a} \), where \( a = b/(b+c) \)

E. Marginal utility

1. extra utility from some extra consumption of one of the goods, holding the other good fixed
2. this is a derivative, but a special kind of derivative — a partial derivative
3. this just means that you look at the derivative of \( u(x_1, x_2) \) keeping \( x_2 \) fixed — treating it like a constant
4. examples
   a) if \( u(x_1, x_2) = x_1 + x_2 \), then \( MU_1 = \frac{\partial u}{\partial x_1} = 1 \)
   b) if \( u(x_1, x_2) = x_1^a x_2^{1-a} \), then \( MU_1 = \frac{\partial u}{\partial x_1} = ax_1^{a-1} x_2^{1-a} \)
5. note that marginal utility depends on which utility function you choose to represent preferences
   a) if you multiply utility times 2, you multiply marginal utility times 2
   b) thus it is not an operational concept
   c) however, \( MU \) is closely related to \( MRS \), which is an operational concept
6. relationship between $MU$ and $MRS$
   a) $u(x_1, x_2) = k$, where $k$ is a constant, describes an indiffer-
      ence curve
   b) we want to measure slope of indifference curve, the $MRS$
   c) so consider a change $(dx_1, dx_2)$ that keeps utility constant.
      Then
      \[
      MU_1 dx_1 + MU_2 dx_2 = 0
      \]
      \[
      \frac{\partial u}{\partial x_1} dx_1 + \frac{\partial u}{\partial x_2} dx_2 = 0
      \]
   d) hence
      \[
      \frac{dx_2}{dx_1} = -\frac{MU_1}{MU_2}
      \]
   e) so we can compute $MRS$ from knowing the utility function

F. Example
1. take a bus or take a car to work?
2. let $x_1$ be the time of taking a car, $y_1$ be the time of taking a
   bus. Let $x_2$ be cost of car, etc.
3. suppose utility function takes linear form $U(x_1, \ldots, x_n) =$
   $\beta_1 x_1 + \ldots + \beta_n x_n$
4. we can observe a number of choices and use statistical tech-
   niques to estimate the parameters $\beta_i$ that best describe choices
5. one study that did this could forecast the actual choice over
   93% of the time
6. once we have the utility function we can do many things with
   it:
   a) calculate the marginal rate of substitution between two
      characteristics
      1) how much money would the average consumer give up
         in order to get a shorter travel time?
   b) forecast consumer response to proposed changes
   c) estimate whether proposed change is worthwhile in a
      benefit-cost sense
Choice

A. Optimal choice

1. move along the budget line until preferred set doesn’t cross the budget set. Figure 5.1.

2. note that tangency occurs at optimal point — necessary condition for optimum. In symbols: $MRS = -\text{price ratio} = -\frac{p_1}{p_2}$.
   a) exception — kinky tastes. Figure 5.2.
   b) exception — boundary optimum. Figure 5.3.

3. tangency is not sufficient. Figure 5.4.
   a) unless indifference curves are convex.
   b) unless optimum is interior.

4. optimal choice is demanded bundle
   a) as we vary prices and income, we get demand functions.
   b) want to study how optimal choice — the demanded bundle — changes as price and income change
B. Examples
1. perfect substitutes: \( x_1 = m/p_1 \) if \( p_1 < p_2 \); 0 otherwise. 

Figure 5.5.
2. perfect complements: \( x_1 = \frac{m}{(p_1 + p_2)} \). Figure 5.6.
3. neutrals and bads: \( x_1 = \frac{m}{p_1} \).
4. discrete goods. Figure 5.7.
   a) suppose good is either consumed or not
   b) then compare \((1, m - p_1)\) with \((0, m)\) and see which is better.
5. concave preferences: similar to perfect substitutes. Note that tangency doesn’t work. Figure 5.8.
6. Cobb-Douglas preferences: \( x_1 = \frac{am}{p_1} \). Note constant budget shares, \( a = \) budget share of good 1.

C. Estimating utility function
1. examine consumption data
2. see if you can “fit” a utility function to it
3. e.g., if income shares are more or less constant, Cobb-Douglas does a good job
4. can use the fitted utility function as guide to policy decisions
5. in real life more complicated forms are used, but basic idea is the same
D. Implications of $MRS$ condition
1. why do we care that $MRS = -\text{price ratio}$?
2. if everyone faces the same prices, then everyone has the same local trade-off between the two goods. This is independent of income and tastes.
3. since everyone locally values the trade-off the same, we can make policy judgments. Is it worth sacrificing one good to get more of the other? Prices serve as a guide to relative marginal valuations.

E. Application — choosing a tax. Which is better, a commodity tax or an income tax?
1. can show an income tax is always better in the sense that given any commodity tax, there is an income tax that makes the consumer better off. Figure 5.9.

2. outline of argument:
   a) original budget constraint: $p_1x_1 + p_2x_2 = m$
   b) budget constraint with tax: $(p_1 + t)x_1 + p_2x_2 = m$
   c) optimal choice with tax: $(p_1 + t)x_1^* + p_2x_2^* = m$
   d) revenue raised is $tx_1^*$
   e) income tax that raises same amount of revenue leads to budget constraint: $p_1x_1 + p_2x_2 = m - tx_1^*$
      1) this line has same slope as original budget line
      2) also passes through $(x_1^*, x_2^*)$
      3) proof: $p_1x_1^* + p_2x_2^* = m - tx_1^*$
4) this means that \((x_1^*, x_2^*)\) is affordable under the income tax, so the optimal choice under the income tax must be even better than \((x_1^*, x_2^*)\)

3. caveats
   a) only applies for one consumer — for each consumer there is an income tax that is better
   b) income is exogenous — if income responds to tax, problems
   c) no supply response — only looked at demand side

F. Appendix — solving for the optimal choice
   1. calculus problem — constrained maximization
   2. max \(u(x_1, x_2)\) s.t. \(p_1 x_1 + p_2 x_2 = m\)
   3. method 1: write down \(MRS = p_1/p_2\) and budget constraint and solve.
   4. method 2: substitute from constraint into objective function and solve.
   5. method 3: Lagrange’s method
      a) write Lagrangian: \(L = u(x_1, x_2) - \lambda (p_1 x_1 + p_2 x_2 - m)\).
      b) differentiate with respect to \(x_1, x_2, \lambda\).
      c) solve equations.
   6. example 1: Cobb-Douglas problem in book
   7. example 2: quasilinear preferences
      a) max \(u(x_1) + x_2\) s.t. \(p_1 x_1 + x_2 = m\)
      b) easiest to substitute, but works each way
Demand

A. Demand functions — relate prices and income to choices

B. How do choices change as economic environment changes?
   1. changes in income
      a) this is a parallel shift out of the budget line
      b) increase in income increases demand — normal good.
      Figure 6.1.

![Figure 6.1](image-url)
c) increase in income decreases demand — \textbf{inferior} good. Figure 6.2.

d) as income changes, the optimal choice moves along the \textbf{income expansion path}
e) the relationship between the optimal choice and income, with prices fixed, is called the **Engel curve**. Figure 6.3.

2. changes in price
   a) this is a tilt or pivot of the budget line
b) decrease in price increases demand — **ordinary** good. Figure 6.9.

c) decrease in price decreases demand — **Giffen** good. Figure 6.10.

d) as price changes the optimal choice moves along the **offer curve**

e) the relationship between the optimal choice and a price, with income and the other price fixed, is called the **demand curve**

C. Examples

1. perfect substitutes. Figure 6.12.
2. perfect complements. Figure 6.13.
3. discrete good. Figure 6.14.
   a) reservation price — price where consumer is just indifferent between consuming next unit of good and not consuming it
   b) \( u(0, m) = u(1, m - r_1) \)
   c) special case: quasilinear preferences
   d) \( v(0) + m = v(1) + m - r_1 \)
   e) assume that \( v(0) = 0 \)
   f) then \( r_1 = v(1) \)
   g) similarly, \( r_2 = v(2) - v(1) \)
   h) reservation prices just measure **marginal utilities**
D. Substitutes and complements

1. increase in $p_2$ increases demand for $x_1$ — substitutes
2. increase in $p_2$ decreases demand for $x_1$ — complements
E. Inverse demand curve
   1. usually think of demand curve as measuring quantity as a function of price — but can also think of price as a function of quantity
   2. this is the inverse demand curve
   3. same relationship, just represented differently
Revealed Preference

A. Motivation
1. up until now we’ve started with preference and then described behavior
2. revealed preference is “working backwards” — start with behavior and describe preferences
3. recovering preferences — how to use observed choices to “estimate” the indifference curves

B. Basic idea
1. if \((x_1, x_2)\) is chosen when \((y_1, y_2)\) is affordable, then we know that \((x_1, x_2)\) is at least as good as \((y_1, y_2)\)
2. in equations: if \((x_1, x_2)\) is chosen when prices are \((p_1, p_2)\) and \(p_1 x_1 + p_2 x_2 \geq p_1 y_1 + p_2 y_2\), then \((x_1, x_2) \succeq (y_1, y_2)\)
3. see Figure 7.1.

![Figure 7.1](image)

4. if \(p_1 x_1 + p_2 x_2 \geq p_1 y_1 + p_2 y_2\), we say that \((x_1, x_2)\) is directly revealed preferred to \((y_1, y_2)\)
5. If $X$ is directly revealed preferred to $Y$, and $Y$ is directly revealed preferred to $Z$ (etc.), then we say that $X$ is **indirectly revealed preferred** to $Z$. See Figure 7.2.
6. the “chains” of revealed preference can give us a lot of information about the preferences. See Figure 7.3.
7. the information revealed about tastes by choices can be used in formulating economic policy

C. Weak Axiom of Revealed Preference
1. recovering preferences makes sense only if consumer is actually maximizing
2. what if we observed a case like Figure 7.4.
3. in this case X is revealed preferred to Y and Y is also revealed preferred to X!
4. in symbols, we have \((x_1, x_2)\) purchased at prices \((p_1, p_2)\) and \((y_1, y_2)\) purchased at prices \((q_1, q_2)\) and \(p_1 x_1 + p_2 x_2 > p_1 y_1 + p_2 y_2\) and \(q_1 y_1 + q_2 y_2 > q_1 x_1 + q_2 x_2\)
5. this kind of behavior is inconsistent with the optimizing model of consumer choice
6. the Weak Axiom of Revealed Preference (WARP) rules out this kind of behavior
7. WARP: if \((x_1, x_2)\) is directly revealed preferred to \((y_1, y_2)\), then \((y_1, y_2)\) cannot be directly revealed preferred to \((x_1, x_2)\)
8. WARP: if \(p_1 x_1 + p_2 x_2 \geq p_1 y_1 + p_2 y_2\), then it must happen that \(q_1 y_1 + q_2 y_2 \leq q_1 x_1 + q_2 x_2\)
9. this condition can be checked by hand or by computer
D. Strong Axiom of Revealed Preference

1. WARP is only a necessary condition for behavior to be consistent with utility maximization

2. Strong Axiom of Revealed Preference (SARP): if \((x_1, x_2)\) is directly or indirectly revealed preferred to \((y_1, y_2)\), then \((y_1, y_2)\) cannot be directly or indirectly revealed preferred to \((x_1, x_2)\)

3. SARP is a necessary and sufficient condition for utility maximization

4. this means that if the consumer is maximizing utility, then his behavior must be consistent with SARP

5. furthermore if his observed behavior is consistent with SARP, then we can always find a utility function that explains the behavior of the consumer as maximizing behavior.

6. can also be tested by a computer

E. Index numbers

1. given consumption and prices in 2 years, base year \(b\) and some other year \(t\)

2. how does consumption in year \(t\) compare with base year consumption?
3. general form of a consumption index:
\[
\frac{w_1x_1^t + w_2x_2^t}{w_1x_1^b + w_2x_2^b}
\]

4. natural to use prices as weights
5. get two indices depending on whether you use period \( t \) or period \( b \) prices
6. Paasche index uses period \( t \) (current period) weights:
\[
\frac{p_1^t x_1^t + p_2^t x_2^t}{p_1^t x_1^b + p_2^t x_2^b}
\]

7. Laspeyres index uses period \( b \) (base period) weights:
\[
\frac{p_1^b x_1^t + p_2^b x_2^t}{p_1^b x_1^b + p_2^b x_2^b}
\]

8. note connection with revealed preference: if Paasche index is greater than 1, then period \( t \) must be better than period \( b \):
   a) \[
   \frac{p_1^t x_1^t + p_2^t x_2^t}{p_1^t x_1^b + p_2^t x_2^b} > 1
   \]
   b) \[
   p_1^t x_1^t + p_2^t x_2^t > p_1^t x_1^b + p_2^t x_2^b
   \]
   c) so period \( t \) is revealed preferred to period \( b \)
9. same sort of thing can be done with Laspeyres index — if Laspeyres index is less than 1, consumer is worse off
Slutsky Equation

A. We want a way to decompose the effect of a price change into “simpler” pieces.
   1. That’s what analysis is all about
   2. Break up into simple pieces to determine behavior of whole

B. Break up price change into a pivot and a shift — see Figure 8.2.

1. These are hypothetical changes
2. We can examine each change in isolation and look at sum of two changes

C. Change in demand due to pivot is the substitution effect.
   1. This measures how demand changes when we change prices, keeping purchasing power fixed
   2. How much would a person demand if he had just enough money to consume the original bundle?
   3. This isolates the pure effect from changing the relative prices
   4. Substitution effect must be negative due to revealed preference.
      a) “Negative” means quantity moves opposite the direction of price
D. Change in demand due to shift is the **income effect**.
   1. increase income, keep prices fixed
   2. income effect can increase or decrease demand depending on whether we have a normal or inferior good

E. Total change in demand is substitution effect plus the income effect.
   1. if good is normal good, the substitution effect and the income effect reinforce each other
   2. if good is inferior good, total effect is ambiguous
   3. see Figure 8.3.

---

**Figure 8.3**

F. Specific examples
   1. perfect complements — Figure 8.4.
   2. perfect substitutes — Figure 8.5.
   3. quasilinear — Figure 8.6.
G. Application — rebating a tax
1. put a tax on gasoline and return the revenues
2. original budget constraint: $px^* + y^* = m$
3. after tax budget constraint: $(p + t)x' + y' = m + tx'$
4. so consumption after tax satisfies $px' + y' = m$
5. so $(x', y')$ was affordable originally and rejected in favor of $(x^*, y^*)$
6. consumer must be worse off

H. Rates of change
1. can also express Slutsky effect in terms of rates of change
2. takes the form
   \[
   \frac{\partial x}{\partial p} = \frac{\partial x^*}{\partial p} - \frac{\partial x}{\partial m} x
   \]
3. can interpret each part just as before
Buying and Selling

A. Up until now, people have only had money to exchange for goods. But in reality, people sell things they own (e.g., labor) to acquire goods. Want to model this idea.

B. Net and gross demands
1. **endowment**: \((\omega_1, \omega_2)\) — what you have before you enter the market.
2. **gross demands**: \((x_1, x_2)\) — what you end up consuming.
3. **net demands**: \((x_1 - \omega_1, x_2 - \omega_2)\) — what you actually buy (positive) and sell (negative).
4. for economists gross demands are more important; for laypeople net demands are more important.

C. Budget constraint
1. value of what you consume = value of what you sell.
2. \(p_1 x_1 + p_2 x_2 = p_1 \omega_1 + p_2 \omega_2\)
3. \(p_1 (x_1 - \omega_1) + p_2 (x_2 - \omega_2) = 0\)
4. budget line depicted in Figure 9.1.

![Figure 9.1](image)

Note endowment is always affordable.

5. with two goods, the consumer is always a net demander of one good, a net supplier of the other.
D. Comparative statics

1. changing the endowment
   a) normal and inferior
   b) increasing the value of the endowment makes the consumer better off. Note that this is different from increasing the value of the consumption bundle. Need access to market.

2. changing prices
   a) if the price of a good the consumer is selling goes down, and the consumer decides to remain a seller, then welfare goes down. See Figure 9.3.

   b) if the consumer is a net buyer of a good and the price decreases, then the consumer will remain a net buyer. Figure 9.4.
   c) etc.

3. offer curves and demand curves
   a) offer curves — what consumer “offers” to buy or sell
   b) gross demand curve
   c) net demand curves (and net supply curves)
E. Slutsky equation

1. when prices change, we now have three effects
   a) ordinary substitution effect
   b) ordinary income effect
   c) endowment income effect — change in the value of the endowment affects demand.
2. three effects shown in Figure 9.7.
3. the income effect depends on the net demand.
4. Slutsky equation now takes the form

   \[ \frac{\partial x_1}{\partial p_1} = \frac{\partial x_1^s}{\partial p_1} + (\omega_1 - x_1) \frac{\partial x_1}{\partial m} \]

5. read through proof in appendix.

F. Labor supply

G. Two goods

1. consumption \( (C) \)
2. labor \( (L) \) — maximum amount you can work is \( \bar{L} \)
3. money \( (M) \)
H. Budget constraint for labor supply
1. \( pC = M + wL \)
2. define \( \bar{C} = M/p \)
3. \( pC + w(\bar{L} - L) = p\bar{C} + w\bar{L} \)
4. define leisure \( R = \bar{L} - L; \) note \( \bar{R} = \bar{L} \)
5. \( pC + wR = p\bar{C} + w\bar{R} = p\bar{C} + w\bar{L} \)
6. this is just like ordinary budget constraint
7. supply of labor is like demand for leisure
8. \( w/p \) is price of leisure

I. Comparative statics
1. apply Slutsky equation to demand for leisure to get

\[
\frac{\partial R}{\partial w} = \text{substitution effect} + (\bar{R} - R) \times \text{income effect}
\]

2. increase in the wage rate has an ambiguous effect on supply of labor. Depends on how much labor is supplied already.
3. backward bending labor supply curve

J. Overtime
1. offer workers a higher straight wage, they may work less.
2. offer them a higher overtime wage, they must work at least as much.
3. overtime is a way to get at the substitution effect.
Intertemporal Choice

A. Budget constraint
1. \((m_1, m_2)\) money in each time period is endowment
2. allow the consumer to borrow and lend at rate \(r\)
3. \(c_2 = m_2 + (1 + r)(m_1 - c_1)\)
4. note that this works for both borrowing and lending, as long as it is at the same interest rate
5. various forms of the budget constraint
   a) \((1 + r)c_1 + c_2 = (1 + r)m_1 + m_2\) — future value
   b) \(c_1 + c_2/(1 + r) = m_1 + m_2/(1 + r)\) — present value
   c) choice of numeraire
   d) see Figure 10.2.

![Figure 10.2](image-url)

6. preferences — convexity and monotonicity are very natural

B. Comparative statics
1. if consumer is initially a lender and interest rate increases, he remains a lender. Figure 10.4.
2. a borrower is made worse off by an increase in the interest rate. Figure 10.5.
3. Slutsky allows us to look at the effect of increasing the price of today’s consumption (increasing the interest rate)
   a) change in consumption today when interest rate increases
      = substitution effect + \((m_1 - c_1)\) income effect
   b) assuming normality, an increase in interest rate lowers current consumption for a borrower, and has an ambiguous effect for lender
   c) provide intuition

C. Inflation
1. put in prices, \(p_1 = 1\) and \(p_2\)
2. budget constraint takes the form
   \[ p_2 c_2 = m_2 + (1 + r)(m_1 - c_1) \]
3. or
   \[ c_2 = \frac{m_2}{p_2} + \frac{(1 + r)}{p_2}(m_1 - c_1) \]
4. if \(\pi\) is rate of inflation, then \(p_2 = (1 + \pi)p_1\)
5. \(1 + \rho = (1 + r)/(1 + \pi)\) is the real interest rate
6. \(\rho = (r - \pi)/(1 + \pi)\) or \(\rho \approx r - \pi\)
D. Present value — a closer look
   1. future value and present value — what do they mean?
   2. if the consumer can borrow and lend freely, then she would always prefer a consumption pattern with a greater present value.

E. Present value works for any number of periods.

F. Use of present value
   1. the one correct way to rank investment decisions
   2. linear operation, so relatively easy to calculate

G. Bonds
   1. coupon $x$, maturity date $T$, face value $F$
   2. consols
   3. the value of a console is given by $PV = x/r$
      a) proof: $x = r \times PV$

H. Installment loans
   1. borrow some money and pay it back over a period of time
   2. what is the true rate of interest?
   3. example: borrow $1,000 and pay back 12 equal installments of $100.
   4. have to value a stream of payments of $1,000, −100, . . . , −100.
   5. turns out that the true interest rate is about 35%!
Asset Markets

A. Consider a world of perfect certainty. Then all assets must have the same rate of return.
1. if one asset had a higher rate of return than another, who would buy the asset with the lower return?
2. how do asset prices adjust? Answer: Riskless arbitrage.
   a) two assets. Bond earns $r$, other asset costs $p_0$ now.
   b) invest $1$ in bond, get $1 + r$ dollars tomorrow.
   c) invest $p_0x = 1$ dollars in other asset, get $p_1x$ dollars tomorrow.
   d) amounts must be equal, which says that $1 + r = p_1/p_0$.
3. this is just another way to say present value.
   a) $p_0 = p_1/(1 + r)$.
4. think about the process of adjustment.

B. Example from stock market
1. index futures and underlying assets that make up the futures.
2. no risk in investment, even though asset values are risky, because there is a fixed relationship between the two assets at the time of expiration.

C. Adjustments for differences in characteristics
1. liquidity and transactions cost
2. taxes
3. form of returns — consumption return and financial return

D. Applications
1. depletable resource — price of oil
   a) let $p_t = \text{price of oil at time } t$
   b) oil in the ground is like money in the bank, so $p_{t+1} = (1 + r)p_t$
   c) demand equals supply over time
   d) let $T = \text{time to exhaustion}$, $D = \text{demand per year}$, and $S = \text{available supply}$. Hence $T = S/D$
   e) let $C = \text{cost of next best alternative (e.g., liquified coal)}$
   f) arbitrage implies $p_0 = C/(1 + r)^T$
2. harvesting a forest
   a) $F(t) = \text{value of forest at time } t$
   b) natural to think of this increasing rapidly at first and then slowing down
   c) harvest when rate of growth of forest = rate of interest.

E. This theory tells you relationships that have to hold between asset prices, given the interest rate.
F. But what determines the interest rate?
   1. answer: aggregate borrowing and lending behavior
   2. or: consumption and investment choices over time

G. What do financial institutions do?
   1. adjust interest rate so that amount people want to borrow equals amount they want to lend
   2. change pattern of consumption possible over time. Example of college student and retiree
   3. example of entrepreneur and investors
Uncertainty

A. Contingent consumption
1. what consumption or wealth you will get in each possible outcome of some random event.
2. example: rain or shine, car is wrecked or not, etc.
3. consumer cares about pattern of contingent consumption: $U(c_1, c_2)$.
4. market allows you to trade patterns of contingent consumption — insurance market. Insurance premium is like a relative price for the different kinds of consumption.
5. can use standard apparatus to analyze choice of contingent consumption.

B. Utility functions
1. preferences over the consumption in different events depend on the probabilities that the events will occur.
2. so $u(c_1, c_2, \pi_1, \pi_2)$ will be the general form of the utility function.
3. under certain plausible assumptions, utility can be written as being linear in the probabilities, $p_1 u(c_1) + p_2 u(c_2)$. That is, the utility of a pattern of consumption is just the expected utility over the possible outcomes.

C. Risk aversion
1. shape of expected utility function describes attitudes towards risk.
2. draw utility of wealth and expected utility of gamble. Note that a person prefers a sure thing to expected value. Figure 12.2.
3. diversification and risk sharing

D. Role of the stock market
1. aids in diversification and in risk sharing.
2. just as entrepreneur can rearrange his consumption patterns through time by going public, he can also rearrange his consumption across states of nature.
Figure 12.2
Risky Assets

A. Utility depends on mean and standard deviation of wealth.
   1. utility = \( u(\mu_w, \sigma_w) \)
   2. this form of utility function describes tastes.

B. Invest in a risky portfolio (with expected return \( r_m \)) and a riskless asset (with return \( r_f \))
   1. suppose you invest a fraction \( x \) in the risky asset
   2. expected return = \( xr_m + (1 - x)r_f \)
   3. standard deviation of return = \( x\sigma_m \)
   4. this relationship gives “budget line” as in Figure 13.2.

![Figure 13.2: Indifference curves and budget line](image)

C. At optimum we must have the price of risk equal to the slope of the budget line: \( MRS = (r_m - r_f)/\sigma_m \)
   1. the observable value \( (r_m - r_f)/\sigma_m \) is the price of risk
   2. can be used to value other investments, like any other price
D. Measuring the risk of a stock — depends on how it contributes to the risk of the overall portfolio.
1. $\beta_i = \text{covariance of asset } i \text{ with the market portfolio/standard deviation of market portfolio}$
2. roughly speaking, $\beta_i$ measures how sensitive a particular asset is to the market as a whole
3. assets with negative betas are worth a lot, since they reduce risk
4. how returns adjust — plot the market line

E. Equilibrium
1. the risk-adjusted rates of return should be equalized
2. in equations:

$$r_i - \beta_i(r_m - r_f) = r_j - \beta_j(r_m - r_f)$$

3. suppose asset $j$ is riskless; then

$$r_i - \beta_i(r_m - r_f) = r_f$$

4. this is called the Capital Asset Pricing Model (CAPM)

F. Examples of use of CAPM
1. how returns adjust — see Figure 13.4.
2. public utility rate of return choice
3. ranking mutual funds
4. investment analysis, public and private
Consumer’s Surplus

A. Basic idea of consumer’s surplus
1. want a measure of how much a person is willing to pay for something. How much a person is willing to sacrifice of one thing to get something else.
2. price measures marginal willingness to pay, so add up over all different outputs to get total willingness to pay.
3. total benefit (or gross consumer’s surplus), net consumer’s surplus, change in consumer’s surplus. See Figure 14.1.

![Figure 14.1: Gross and Net Surplus](image)

B. Discrete demand
1. remember that the reservation prices measure the “marginal utility”
2. \( r_1 = v(1) - v(0), \) \( r_2 = v(2) - v(1), \) \( r_3 = v(3) - v(2), \) etc.
3. hence, \( r_1 + r_2 + r_3 = v(3) - v(0) = v(3) \) (since \( v(0) = 0)\)
4. this is just the total area under the demand curve.
5. in general to get the “net” utility, or net consumer’s surplus, have to subtract the amount that the consumer has to spend to get these benefits
C. Continuous demand. Figure 14.2.
1. suppose utility has form \( v(x) + y \)
2. then inverse demand curve has form \( p(x) = v'(x) \)
3. by fundamental theorem of calculus:

\[
v(x) - v(0) = \int_0^x v'(t) \, dt = \int_0^x p(t) \, dt
\]

4. This is the generalization of discrete argument

D. Change in consumer’s surplus. Figure 14.3.

E. Producer’s surplus — area above supply curve. Change in producer’s surplus
1. see Figure 14.6.
2. intuitive interpretation: the sum of the marginal willingnesses to supply

F. This all works fine in the case of quasilinear utility, but what do you do in general?
G. Compensating and equivalent variation. See Figure 14.4.
1. compensating: how much extra money would you need after a price change to be as well off as you were before the price change?

2. equivalent: how much extra money would you need before the price change to be just as well off as you would be after the price change?

3. in the case of quasilinear utility, these two numbers are just equal to the change in consumer’s surplus.

4. in general, they are different . . . but the change in consumer’s surplus is usually a good approximation to them.
Market Demand

A. To get market demand, just add up individual demands.
   1. add horizontally
   2. properly account for zero demands; Figure 15.2.

B. Often think of market behaving like a single individual.
   1. **representative consumer** model
   2. not true in general, but reasonable assumption for this course

C. Inverse of aggregate demand curve measures the $MRS$ for each individual.

D. Reservation price model
   1. appropriate when one good comes in large discrete units
   2. reservation price is price that just makes a person indifferent
   3. defined by $u(0, m) = u(1, m - p_1^*)$
   4. see Figure 15.3.
   5. add up demand curves to get aggregate demand curve

---

**Figure 15.2**

A. Agent 1's demand
   \[ D_1(p_1) \]

B. Agent 2's demand
   \[ D_2(p_2) \]

C. Market demand = sum of the two demand curves
   \[ D_1(p_1) + D_2(p_2) \]
E. Elasticity

1. measures responsiveness of demand to price

2. 
   \[ \epsilon = \frac{p \frac{dq}{dp}}{q} \]

3. example for linear demand curve
   a) for linear demand, \( q = a - bp \), so \( \epsilon = -bp/q = -bp/(a-bp) \)
   b) note that \( \epsilon = -1 \) when we are halfway down the demand curve
   c) see Figure 15.4.

4. suppose demand takes form \( q = Ap^{-b} \)

5. then elasticity is given by
   \[ \epsilon = -\frac{p}{q} b Ap^{-b-1} = -\frac{-bAp^{-b}}{Ap^{-b}} = -b \]

6. thus elasticity is constant along this demand curve

7. note that \( \log q = \log A - b \log p \)

8. what does elasticity depend on? In general how many and how close substitutes a good has.
F. How does revenue change when you change price?
1. \( R = pq \), so \( \Delta R = (p + dp)(q + dq) - pq = pdq + qdp + dpdq \)
2. last term is very small relative to others
3. \( dR/dp = q + p dq/dp \)
4. see Figure 15.5.
5. \( dR/dp > 0 \) when \( |e| < 1 \)

G. How does revenue change as you change quantity?
1. marginal revenue = \( MR = dR/dq = p + q dp/dq = p[1 + 1/e] \).
2. **elastic**: absolute value of elasticity greater than 1
3. **inelastic**: absolute value of elasticity less than 1
4. application: Monopolist never sets a price where \( |e| < 1 \) — because it could always make more money by reducing output.

H. Marginal revenue curve
1. always the case that \( dR/dq = p + q dp/dq \).
2. in case of linear (inverse) demand, \( p = a - bq \), \( MR = dR/dq = p - bq = (a - bq) - bq = a - 2bq \).
I. Laffer curve
1. how does tax revenue respond to changes in tax rates?
2. idea of Laffer curve: Figure 15.8.
3. theory is OK, but what do the magnitudes have to be?
4. model of labor market, Figure 15.9.
5. tax revenue = \( T = t\bar{w}S(w(t)) \) where \( w(t) = (1 - t)\bar{w} \)
6. when is \( dT/dt < 0? \)
7. calculate derivative to find that Laffer curve will have negative slope when
   \[
   \frac{dS}{dw} \frac{w}{S} > \frac{1 - t}{t}
   \]
8. so if tax rate is .50, would need labor supply elasticity greater than 1 to get Laffer effect
9. very unlikely to see magnitude this large
Figure 15.8

Figure 15.9
Equilibrium

A. Supply curves — measure amount the supplier wants to supply at each price
1. review idea of net supply from Chapter 9

B. Equilibrium
1. competitive market — each agent takes prices as outside his or her control
   a) many small agents
   b) a few agents who think that the others keep fixed prices
2. equilibrium price — that price where desired demand equals desired supply
   a) \( D(p) = S(p) \)
3. special cases — Figure 16.1.

Figure 16.1

a) vertical supply — quantity determined by supply, price determined by demand
b) horizontal supply — quantity determined by demand, price determined by supply
4. an equivalent definition of equilibrium: where inverse demand curve crosses inverse supply curve
   a) \( P_d(q) = P_s(q) \)
5. examples with linear curves
C. Comparative statics
   1. shift each curve separately
   2. shift both curves together

D. Taxes — nice example of comparative statics
   1. demand price and supply price — different in case of taxes
   2. $p_d = p_s + t$
   3. equilibrium happens when $D(p_d) = S(p_s)$
   4. put equations together:
      a) $D(p_s + t) = S(p_s)$
      b) or $D(p_d) = S(p_d - t)$
   5. also can solve using inverse demands:
      a) $P_d(q) = P_s(q) + t$
      b) or $P_d(q) - t = P_s(q)$
   6. see Figure 16.3.

![Figure 16.3](image)

and Figure 16.4.

E. Passing along a tax — Figure 16.5.
   1. flat supply curve
   2. vertical supply curve
F. Deadweight loss of a tax — Figure 16.7.
1. benefits to consumers
2. benefits to producers
3. value of lost output

G. Market for loans
1. tax system subsidizes borrowing, tax lending
2. with no tax: \( D(r^*) = S(r^*) \)
3. with tax: \( D((1 - t)r') = S((1 - t)r') \)
4. hence, \((1 - t)r' = r^*\). Quantity transacted is same
5. see Figure 16.8.

H. Food subsidies
1. buy up harvest and resell at half price.
2. before program: \( D(p^*) + K = S \)
3. after program: \( D(\hat{p}/2) + K = S \)
4. so, \( \hat{p} = 2p^* \).
5. subsidized mortgages — unless the housing stock changes, no effect on cost.

I. Pareto efficiency
1. efficient output is where demand equals supply
2. because that is where demand price equals supply price.
3. that is, the marginal willingness to buy equals the marginal willingness to sell.
4. deadweight loss measures loss due to inefficiency.
Figure 16.8
Auctions

A. Auctions are one of the oldest form of markets
   1. 500 BC in Babylon
   2. 1970s offshore oil
   3. 1990s FCC airwave auctions
   4. various privatization projects

B. Classification of auctions
   1. private-value auctions
   2. common-value auctions

C. Bidding rules
   1. English auction, reserve price, bid increment
   2. Dutch auction
   3. sealed-bid auction
   4. Vickrey auction (philatelist auction, second-price auction)

D. Auction design
   1. special case of economic mechanism design
   2. possible goals
      a) Pareto efficiency
      b) profit maximization
   3. Pareto efficiency in private value auction
      a) person who values the good most highly gets it
      b) otherwise would be Pareto improvement possible
   4. Case 1: seller knows values $v_1, \ldots, v_n$
      a) trivial answer: set price at highest value
      b) this is Pareto efficient
   5. Case 2: seller doesn’t know value
      a) run English auction
      b) person with highest value gets the good
      c) Pareto efficient
      d) pays price equal to second-highest value
   6. profit maximization in private-value auctions
      a) depends on sellers’ beliefs about buyers’ values
      b) example: 2 bidders with values of either $10$ or $100$
      c) assume equally likely so possibilities are $(10,10)$, $(10,100)$, $(100,10)$, or $(100,100)$
      d) minimal bid increment of $1$, flip a coin for ties
      e) revenue will be $(10,11,11,100)$
      f) expected revenue will be $33$
      g) is this the best the seller can do?
      h) No! If he sets a reserve price of $100$ he gets $(0,100,100,100)$
      i) expected profit is $75$ which is much better
      j) *not* Pareto efficient
7. Dutch auction, sealed-bid auction  
   a) might not be Pareto efficient

8. Vickrey auction  
   a) if everyone reveals true value will be efficient 
   b) but will they want to tell the truth?  
   c) Yes! Look at special case of two buyers  
   d) payoff = \( \text{Prob}(b_1 \geq b_2)[v_1 - b_2] \)  
   e) if \( v_1 > b_2 \), want to make probability = 1  
   f) if \( v_1 < b_2 \), want to make probability = 0  
   g) it pays to tell the truth (in this case)  
   h) note that this is essentially the same outcome as English auction

E. Problems with auctions  
   1. susceptible to collusion (bidding rings)  
   2. dropping out (Australian satellite-TV licenses)

F. Winner’s curse  
   1. common value auction  
   2. assume that each person bids estimated value  
   3. then most optimistic bidder wins  
   4. but this is almost certainly an overestimate of value  
   5. optimal strategy is to adjust bid downward  
   6. amount that you adjust down depends on number of other bidders
Technology

A. Need a way to describe the technological constraints facing a firm
   1. what patterns of inputs and outputs are feasible?

B. Inputs
   1. factors of production
   2. classifications: labor, land, raw materials, capital
   3. usually try to measure in flows
   4. financial capital vs. physical capital

C. Describing technological constraints
   1. production set — combinations of inputs and outputs that are feasible patterns of production
   2. production function — upper boundary of production set
   3. see Figure 17.1.

4. isoquants — all combinations of inputs that produce a constant level of output
5. isoquants (constant output) are just like indifference curves (constant utility)
D. Examples of isoquants
1. fixed proportions — one man, one shovel
2. perfect substitutes — pencils
3. Cobb-Douglas — $y = Ax_1^a x_2^b$
4. can’t take monotonic transformations any more!

E. Well-behaved technologies
1. monotonic — more inputs produce more output
2. convex — averages produce more than extremes

F. Marginal product
1. $MP_1$ is how much extra output you get from increasing the input of good 1
2. holding good 2 fixed
3. $MP_1 = \frac{\partial f(x_1, x_2)}{\partial x_1}$

G. Technical rate of substitution
1. like the marginal rate of substitution
2. given by the ratio of marginal products
3. $\text{TRS} = \frac{dx_2}{dx_1} = -\frac{\partial f/\partial x_1}{\partial f/\partial x_2}$

H. Diminishing marginal product
1. more and more of a single input produces more output, but at a decreasing rate. See Figure 17.5.
2. law of diminishing returns

I. Diminishing technical rate of substitution
1. equivalent to convexity
2. note difference between diminishing $MP$ and diminishing $\text{TRS}$

J. Long run and short run
1. All factors varied — long run
2. Some factors fixed — short run

K. Returns to scale
1. constant returns — baseline case
2. increasing returns
3. decreasing returns
Figure 17.5

\[ y = f(x_1, R_2) \]
Profit Maximization

A. Profits defined to be revenues minus costs
   1. value each output and input at its market price — even if it is not sold on a market.
   2. it could be sold, so using it in production rather than somewhere else is an opportunity cost.
   3. measure in terms of flows. In general, maximize present value of flow of profits.

B. Stock market value
   1. in world of certainty, stock market value equals present value of stream of profits
   2. so maximizing stock market value is the same as maximizing present value of profits
   3. uncertainty — more complicated, but still works

C. Short-run and long-run maximization
   1. fixed factors — plant and equipment
   2. quasi-fixed factors — can be eliminated if operate at zero output (advertising, lights, heat, etc.)

D. Short-run profit maximization. Figure 18.1.

\[ \max \ p f(x) - w x \]
2. \( pf'(x^*) - w = 0 \)
3. in words: the value of the marginal product equals wage rate
4. comparative statics: change \( w \) and \( p \) and see how \( x \) and \( f(x) \) respond

E. Long-run profit maximization
1. \( p \frac{\partial f}{\partial x_1} = w_1, \ p \frac{\partial f}{\partial x_2} = w_2 \)

F. Profit maximization and returns to scale
1. constant returns to scale implies profits are zero
   a) note that this doesn’t mean that economic factors aren’t all appropriately rewarded
   b) use examples
2. increasing returns to scale implies competitive model doesn’t make sense

G. Revealed profitability
1. simple, rigorous way to do comparative statics
2. observe two choices, at time \( t \) and time \( s \)
3. \( (p^t, w^t, y^t, x^t) \) and \( (p^s, w^s, y^s, x^s) \)
4. if firm is profit maximizing, then must have
\[
\begin{align*}
p^t y^t - w^t x^t & \geq p^t y^s - w^t x^s \\
p^s y^s - w^s x^s & \geq p^s y^t - w^s x^t
\end{align*}
\]
5. write these equations as
\[
\begin{align*}
p^t y^t - w^t x^t & \geq p^t y^s - w^t x^s \\
-p^s y^t + w^s x^t & \geq -p^s y^s + w^s x^s
\end{align*}
\]
6. add these two inequalities:
\[
(p^t - p^s)y^t - (w^t - w^s)x^t \geq (p^t - p^s)y^s - (w^t - w^s)x^s
\]
7. rearrange:
\[
(p^t - p^s)(y^t - y^s) - (w^t - w^s)(x^t - x^s) \geq 0
\]
8. or
\[
\Delta p \Delta y - \Delta w \Delta x \geq 0
\]
9. implications for changing output and factor prices
Cost Minimization

A. Cost minimization problem
1. minimize cost to produce some given level of output:

$$\min_{x_1, x_2} \ w_1 x_1 + w_2 x_2$$

s.t. $$f(x_1, x_2) = y$$

2. geometric solution: slope of isoquant equals slope of isocost curve. Figure 19.1.

3. equation is: $$w_1/w_2 = MP_1/MP_2$$
4. optimal choices of factors are the conditional factor demand functions
5. optimal cost is the cost function
6. examples
   a) if $$f(x_1, x_2) = x_1 + x_2$$, then $$c(w_1, w_2, y) = \min\{w_1, w_2\} y$$
   b) if $$f(x_1, x_2) = \min\{x_1, x_2\}$$, then $$c(w_1, w_2, y) = (w_1 + w_2) y$$
   c) can calculate other answers using calculus
B. Revealed cost minimization

1. suppose we hold output fixed and observe choices at different factor prices.
2. when prices are \((w^s_1, w^s_2)\), choice is \((x^s_1, x^s_2)\), and when prices are \((w^t_1, w^t_2)\), choice is \((x^t_1, x^t_2)\).
3. if choices minimize cost, then we must have

\[
\begin{align*}
    w^t_1 x^t_1 + w^t_2 x^t_2 &\leq w^t_1 x^s_1 + w^t_1 x^s_2 \\
    w^s_1 x^s_1 + w^s_2 x^s_2 &\leq w^s_1 x^t_1 + w^s_2 x^t_2
\end{align*}
\]

4. this is the **Weak Axiom of Cost Minimization (WACM)**
5. what does it imply about firm behavior?
6. multiply the second equation by \(-1\) and get

\[
\begin{align*}
    w^t_1 x^t_1 + w^t_2 x^t_2 &\leq w^t_1 x^s_1 + w^t_1 x^s_2 \\
    -w^s_1 x^t_1 - w^s_2 x^t_2 &\leq -w^s_1 x^s_1 - w^s_2 x^s_2
\end{align*}
\]

7. add these two inequalities:

\[(w^t_1 - w^s_1)(x^t_1 - x^s_1) + (w^t_2 - w^s_2)(x^t_2 - x^s_2) \leq 0\]

\[
\Delta w_1 \Delta x_1 + \Delta w_2 \Delta x_2 \leq 0
\]

8. roughly speaking, “factor demands move opposite to changes in factor prices”
9. in particular, factor demand curves must slope downward.

C. Returns to scale and the cost function

1. increasing returns to scale implies decreasing \(AC\)
2. constant returns implies constant \(AC\)
3. decreasing returns implies increasing \(AC\)

D. Long-run and short-run costs

1. long run: all inputs variable
2. short run: some inputs fixed

E. Fixed and quasi-fixed costs

1. fixed: must be paid, whatever the output level
2. quasi-fixed: only paid when output is positive (heating, lighting, etc.)
Cost Curves

A. Family of cost curves

1. total cost: \( c(y) = c_v(y) + F \)

2. \[
\frac{c(y)}{y} = \frac{c_v(y)}{y} + \frac{F}{y} \\
AC = AVC + AFC
\]

3. see Figure 20.1.

4. marginal cost is the change in cost due to change in output
\( c'(y) = \frac{dc(y)}{dy} = \frac{dc_v(y)}{dy} \)

a) marginal cost equals \( AVC \) at zero units of output
b) goes through minimum point of \( AC \) and \( AVC \). Figure 20.2.

\[
\frac{d}{dy} \frac{c(y)}{y} = \frac{yc'(y) - c(y)}{y^2}
\]

2) this is negative (for example) when \( c'(y) < \frac{c(y)}{y} \)
c) fundamental theorem of calculus implies that

\[ c_v(y) = \int_0^y c'(t) \, dt \]

d) geometrically: the area under the marginal cost curve gives the total variable costs. Figure 20.3.

e) intuitively: the marginal cost curve measures the cost of each additional unit, so adding up the MCs gives the variable cost

B. Example: \( c(y) = y^2 + 1 \)
1. \( AC = y + 1/y \)
2. \( AVC = y \)
3. \( MC = 2y \)
4. Figure 20.4.

C. Long-run cost from short-run cost
1. average costs: Figure 20.8.
2. marginal costs: Figure 20.9.
Figure 20.4
Firm Supply

A. Firms face two sorts of constraints
   1. technological constraints — summarize in cost function
   2. market constraints — how will consumers and other firms react to a given firm’s choice?

B. Pure competition
   1. formally — takes market price as given, outside of any particular firm’s control
   2. example: many small price takers
   3. demand curve facing a competitive firm — Figure 21.1.

C. Supply decision of competitive firm
   1. \( \max_y py - c(y) \)
   2. first-order condition: \( p = c'(y) \)
   3. price equals marginal cost determines supply as function of price
   4. second-order condition: \( -c''(y) \leq 0 \), or \( c'(y) \geq 0 \).
   5. only upward-sloping part of marginal cost curve matters
   6. is it profitable to operate at all?
      a) compare \( py - c_v(y) - F \) with \(-F\)
      b) profits from operating will be greater when \( p > c_v(y)/y \)
      c) operate when price covers average variable costs
D. So supply curve is the upward-sloping part of $MC$ curve that lies above the $AVC$ curve
1. see Figure 21.3.

![Figure 21.3](image)

E. Inverse supply curve
1. $p = c'(y)$ measures the marginal cost curve directly

F. Example: $c(y) = y^2 + 1$
1. $p = 2y$ gives the (inverse) supply curve
2. is $p \geq AVC$?
   a) yes, since $2y \geq y$ for all $y \geq 0$
3. see Figure 21.7.

G. Producer’s surplus
1. producer’s surplus is defined to be $py - c_v(y)$
2. since $c_v(y) = \text{area under marginal cost curve}$
3. producer’s surplus is also the area above the marginal cost curve
4. we can also use the “rectangle” for part of $PS$ and the “area above $MC$” for the rest
5. see Figure 21.5.

H. Long-run supply — use long-run $MC$. In long run, price must be greater than $AC$
I. Special case — constant average cost (CRS): flat supply curve
1. see Figure 21.10.
Figure 21.10
Industry Supply

A. Short-run industry supply
1. sum of the $MC$ curves
2. equilibrium in short run
   a) look for point where $D(p) = S(p)$
   b) can then measure profits of firms
   c) see Figure 22.2.

B. Long-run industry supply
1. change to long-run technology
2. entry and exit by firms
   a) look at curves with different number of firms
   b) find lowest curve consistent with nonnegative profits
   c) see Figure 22.3.

C. Long-run supply curve
1. exact — see Figure 22.4.
2. approximate — flat at $p = \text{minimum } AC$
3. like replication argument
D. Taxation in long and short runs
1. see Figure 22.6.
2. in industry with entry and exit
3. part of tax is borne by each side
4. long run — all borne by consumers

E. Meaning of zero profits
1. pure economic profit means anyone can get it
2. a mature industry may show accounting profits, but economic profits are probably zero

F. Economic rent
1. what if some factors are scarce in the long run?
   a) licenses — liquor, taxicab
   b) raw materials, land, etc.
2. fixed from viewpoint of industry, variable from viewpoint of firm
3. in this case, industry can only support a certain number of firms
4. whatever factor is preventing entry earns rents
   a) always the possibility of entry that drives profits to zero
   b) if profits are being made, firms enter industry by
      1) bringing in new resources
      2) bidding up prices of existing resources
5. see Figure 22.7.
6. discount flow of rents to get asset value
7. politics of rent
   a) rents are a pure surplus payment
   b) but people compete for those rents
   c) taxicab licenses — current holders want very much to prevent entry
   d) subsidies and rents — incidence of subsidy falls on the rents
      1) tobacco subsidies
      2) farm policy in general
   e) rent seeking

G. Energy policy
   1. two-tiered oil pricing
   2. price controls
   3. entitlement program
Monopoly

A. Profit maximization
1. max \( r(y) - c(y) \) implies \( r'(y) = c'(y) \)
2. max \( p(y)y - c(y) \) implies \( p(y) + p'(y)y = c'(y) \)
3. can also write this as
   \[
   p(y) \left[ 1 + \frac{dp}{dy} \frac{y}{p} \right] = c'(y)
   \]
4. or \( p(y)[1 + 1/\epsilon] = c'(y) \)
5. linear case
   a) in case of linear demand, \( p = a - by \), marginal revenue is given by \( MR = a - 2by \)
   b) see Figure 23.1.

![](monopoly_diagram.png)

Figure 23.1

6. constant elasticity, \( q = Ap^\epsilon \)
   a) in this case, \( MR = p[1 + 1/\epsilon] \)
   b) so, optimal condition is \( p[1 + 1/\epsilon] = c'(y) \)
   c) markup on marginal cost
   d) see Figure 23.2.

B. Taxes
1. linear case — price goes up by half of tax. Figure 23.3.
2. log case — price goes up by more than tax, since price is a markup on \( MC \)
C. Inefficiency of monopoly
1. Pareto efficient means no way to make some group better off without hurting some other group
2. Pareto *inefficient* means that there *is* some way to make some group better off without hurting some other group
3. monopoly is Pareto inefficient since \( P > MC \)
4. measure of the deadweight loss — value of lost output
5. see Figure 23.5.

---

**Figure 23.5**

---

D. Patents
1. sometimes we want to pay this cost of inefficiency
2. patents: trade-off of innovation against monopoly losses

E. Natural monopoly
1. public utilities (gas, electricity, telephone) are often thought of as **natural monopolies**
2. occurs when \( p = mc \) is unprofitable — decreasing \( AC \)
3. Figure 23.6.
4. often occurs when fixed costs are big and marginal costs are small
5. how to handle
   a) government operates and covers deficit from general revenues
   b) regulates pricing behavior so that \( price = AC \)
F. Cause of monopoly
   1. \( MES \) large relative to size of market
   2. collusion
   3. law (oranges, sports, etc.)
   4. trademarks, copyrights, brand names, etc.
Monopoly Behavior

A. Price discrimination
   1. first degree — perfect price discrimination
      a) gives Pareto efficient output
      b) same as take-it-or-leave-it offer
      c) producer gets all surplus
   2. second degree — nonlinear pricing
      a) two demand curves
      b) would like to charge each full surplus
      c) but have to charge bigger one less to ensure self-selection
      d) but then want to reduce the amount offered to smaller consumer
   3. third degree — most common
      a) 
      \[
      \max p_1(y_1)y_1 + p_2(y_2)y_2 - c(y_1 + y_2)
      \]
      b) gives us the first-order conditions
      \[
      p_1 + p_1'(y_1)y_1 = c'(y_1 + y_2) \\
      p_2 + p_2'(y_2)y_2 = c'(y_1 + y_2)
      \]
      c) or
      \[
      p_1 \left[1 - \frac{1}{|\epsilon_1|}\right] = MC \\
      p_2 \left[1 - \frac{1}{|\epsilon_2|}\right] = MC
      \]
      d) result: if \( p_1 > p_2 \), then \(|\epsilon_1| < |\epsilon_2|\)
      e) more elastic users pay lower prices

B. Two-part tariffs
   1. what happens if everyone is the same?
   2. entrance fee = full surplus
   3. usage fee = marginal cost

C. Bundling
   1. type A: wtp $120 for word processor, $100 for spreadsheet
   2. type B: wtp $100 for word processor, $120 for spreadsheet
   3. no bundling profits = $400
   4. bundling profits = $440
   5. reduce dispersion of wtp
D. Monopolistic competition

1. rare to see pure monopoly
2. product differentiation – so some market power
3. free entry
4. result — excess capacity theorem
   a) see Figure 24.3.
   b) (but is it really?)
5. location model of product differentiation
   a) ice cream vendors on the boardwalk
   b) socially optimal to locate at 1/4 and 3/4
   c) but this is “unstable”
   d) only stable configuration is for both to locate at middle
   e) is there too much conformity in differentiated markets?
Factor Markets

A. Monopoly in output market
1. marginal product, $MP_x$
2. marginal revenue, $MR_y$
3. marginal revenue product, $MRP_x$
4. value of the marginal product, $pMP_x$
   a) $MRP = p \left[ 1 - \frac{1}{|\epsilon|} \right]$
   b) note that this is less than value of $MP$

B. Monopoly/monoposony in input market
1. market power by demander of factor
2. maximize $pf(x) - w(x)x$
3. get $MR = MC$, but with particular form
4. now $MC = w \left[ 1 + \frac{1}{\epsilon} \right]$
5. linear example: Figure 25.2.

6. minimum wage
C. Upstream and downstream monopoly
   1. one monopolist produces a factor that he sells to another monopolist
   2. suppose that one unit of the input produces one unit of output in downstream monopolist
   3. each monopolist wants to mark up its output price over marginal cost
   4. results in a **double markup**
   5. if firms integrated, would only have a single markup
   6. price would go down
Oligopoly

A. Oligopoly is the study of the interaction of a small number of firms
   1. duopoly is simplest case
   2. unlikely to have a general solution; depends on market structure and specific details of how firms interact

B. Classification of theories
   1. non-collusive
      a) sequential moves
         1) quantity setting — Stackelberg
         2) price setting — price leader
      b) simultaneous moves
         1) quantity setting — Cournot
         2) price setting — Bertrand
   2. collusive

C. Stackelberg behavior
   1. asymmetry — one firm, quantity leader, gets to set quantity first
   2. maximize profits, given the reaction behavior of the other firm
   3. take into response that the other firm will follow my lead
   4. analyze in reverse
   5. firm 2
      a) \( \max_{y_2} P(y_1 + y_2)y_2 - c(y_2) \)
      b) FOC: \( P(y_1 + y_2) + P'(y_1 + y_2)y_2 = c'(y_2) \)
      c) solution gives \textbf{reaction function}, \( f_2(y_1) \)
   6. firm 1
      a) \( \max_{y_1} P(y_1 + f_2(y_1))y_1 - c(y_1) \)
      b) FOC: \( P(y_1 + f_2(y_1)) + P'(y_1 + f_2(y_1))y_1 = c'(y_1) \)
      c) see Figure 26.2.
   7. graphical solution in Figure 26.4.

D. Price-setting behavior
   1. leader sets price, follower takes it as given
   2. given \( p_1 \), firm 2 supplies \( S_2(p_1) \)
   3. if demand is \( D(p) \), this leaves \( D(p_1) - S_2(p_1) \) for leader
   4. hence leader wants to maximize \( p_1y_1 - c(y_1) \) such that \( y_1 = D(p_1) - S_2(p_1) \)
   5. leader faces “residual demand curve”
E. Cournot equilibrium — simultaneous quantity setting
1. each firm makes a choice of output, given its forecast of the other firm’s output
2. let $y_1$ be the output choice of firm 1 and $y_2^e$ be firm 1’s beliefs about firm 2’s output choice
3. maximization problem $\max_{y_1} \ p(y_1 + y_2^e) - c(y_1)$
4. let $Y = y_1 + y_2^e$
5. first-order condition is

$$p(Y) + p'(Y)y_1 = c'(y_1)$$

6. this gives firm 1’s reaction curve — how it chooses output given its beliefs about firm 2’s output
7. see Figure 26.1.

8. look for Cournot equilibrium — where each firm finds its expectations confirmed in equilibrium
9. so $y_1 = y_1^e$ and $y_2 = y_2^e$

F. Example of Cournot
1. assume zero costs
2. linear demand function $p(Y) = a - bY$
3. profit function: $[a - b(y_1 + y_2)]y_1 = ay_1 - by_1^2 - by_1 y_2$
4. derive reaction curve
   a) maximize profits
   b) $a - 2by_1 - by_2 = 0$
   c) calculate to get $y_1 = (a - by_2)/2b$
   d) do same sort of thing to get reaction curve for other firm
5. look for intersection of reaction curves
G. Bertrand – simultaneous price setting
1. consider case with constant identical marginal cost
2. if firm 1 thinks that other firm will set $p_2$, what should it set?
3. if I think $p_2$ is greater than my $MC$, set $p_1$ slightly smaller than $p_2$
4. I get all the customers and make positive profits
5. only consistent (equilibrium) beliefs are $p_1 = p_2 = MC$

H. Collusion
1. firms get together to maximize joint profits
2. marginal impact on joint profits from selling output of either firm must be the same
3. $\max p(y_1 + y_2)[y_1 + y_2] - c(y_1) - c(y_2)$
4. $P(y_1 + y_2) + P'(y_1 + y_2)[y_1 + y_2] = c'(y_1) = c'(y_2)$
5. note instability — if firm 1 believes firm 2 will keep its output fixed, it will always pay it to increase its own output
6. problems with OPEC
7. if it doesn’t believe other firm will keep its output fixed, it will cheat first!
Game Theory

A. Game theory studies strategic interaction, developed by von Neumann and Morgenstern around 1950

B. How to depict payoffs of game from different strategies
   1. two players
   2. two strategies
   3. example

   \[
   \begin{array}{c|cc}
   \text{Row} & \text{Top} & \text{Bottom} \\
   \hline
   \text{Column} & \text{Left} & \text{Right} \\
   \hline
   \text{Top} & 1, 2 & 0, 1 \\
   \text{Bottom} & 2, 1 & 1, 0 \\
   \end{array}
   \]

   a) this depicts a **dominant strategy**
   b) each person has a strategy that is best no matter what the other person does
   c) nice when it happens, but doesn’t happen that often

C. Nash equilibrium
   1. what if there is no dominant strategy?
   2. in this case, look for strategy that is best if the other player plays his best strategy
   3. note the “circularity” of definition
   4. appropriate when you are playing against a “rational” opponent
   5. each person is playing the best given his expectations about the other person’s play and expectations are actually confirmed
   6. example

   \[
   \begin{array}{c|cc}
   \text{Row} & \text{Top} & \text{Bottom} \\
   \hline
   \text{Column} & \text{Left} & \text{Right} \\
   \hline
   \text{Top} & 2, 1 & 0, 0 \\
   \text{Bottom} & 0, 0 & 1, 2 \\
   \end{array}
   \]

   a) note (top, left) is Nash; (bottom, right) is also Nash
7. Nash equilibrium in pure strategies may not exist.

\[
\begin{array}{c|cc}
\text{Row} & \text{Top} & \text{Bottom} \\
\hline
\text{Left} & (0,0) & (1,0) \\
\text{Right} & (0,-1) & (-1,3)
\end{array}
\]

8. but if allow mixed strategies (and people only care about expected payoff), then Nash equilibrium will always exist

D. Prisoner’s dilemma
1. 2 prisoners, each may confess (and implicate other) or deny
2. gives payoff matrix

\[
\begin{array}{c|cc}
\text{Row} & \text{Top} & \text{Bottom} \\
\hline
\text{Left} & (-3,-3) & (0,-6) \\
\text{Right} & (-6,0) & (-1,-1)
\end{array}
\]

3. note that (confess, confess) is unique dominant strategy equilibrium, but (deny, deny) is Pareto efficient
4. example: cheating in a cartel
5. example: agreeing to get rid of spies
6. problem — no way to communicate and make binding agreements

E. Repeated games
1. if game is repeated with same players, then there may be ways to enforce a better solution to prisoner’s dilemma
2. suppose PD is repeated 10 times and people know it
   a) then backward induction says it is a dominant strategy to cheat every round
3. suppose that PD is repeated an indefinite number of times
   a) then may pay to cooperate
4. Axelrod’s experiment: tit-for-tat

F. Example – enforcing cartel and price wars

G. Sequential game — time of choices matters
H. Example:

\[\begin{array}{c|cc}
\text{Row} & \text{Top} & \text{Bottom} \\
\hline
\text{Left} & 1,9 & 1,9 \\
\text{Right} & 0,0 & 2,1 \\
\end{array}\]

1. (Top, Left) and (Bottom, Right) are both Nash equilibria
2. but in extensive form (Top, Left) is not reasonable. Figure 27.6.

3. to solve game, start at end and work backward
4. (Top, Left) is not an equilibrium, since the choice of Top is not a credible choice

I. Example: entry deterrence
   1. stay out and fight
   2. excess capacity to prevent entry — change payoffs
   3. see Figure 27.7.
   4. strategic inefficiency
Figure 27.7
Game Theory

A. Game theory studies strategic interaction, developed by von Neumann and Morgenstern around 1950

B. How to depict payoffs of game from different strategies
   1. two players
   2. two strategies
   3. example

<table>
<thead>
<tr>
<th></th>
<th>Column</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Left</td>
</tr>
<tr>
<td>Top</td>
<td>1, 2</td>
</tr>
<tr>
<td>Bottom</td>
<td>2, 1</td>
</tr>
</tbody>
</table>

   a) this depicts a dominant strategy
   b) each person has a strategy that is best no matter what the other person does
   c) nice when it happens, but doesn’t happen that often

C. Nash equilibrium
   1. what if there is no dominant strategy?
   2. in this case, look for strategy that is best if the other player plays his best strategy
   3. note the “circularity” of definition
   4. appropriate when you are playing against a “rational” opponent
   5. each person is playing the best given his expectations about the other person’s play and expectations are actually confirmed
   6. example

<table>
<thead>
<tr>
<th></th>
<th>Column</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Left</td>
</tr>
<tr>
<td>Top</td>
<td>2, 1</td>
</tr>
<tr>
<td>Bottom</td>
<td>0, 0</td>
</tr>
</tbody>
</table>

   a) note (top, left) is Nash; (bottom, right) is also Nash
7. Nash equilibrium in pure strategies may not exist.

<table>
<thead>
<tr>
<th>Row</th>
<th>Top</th>
<th>Bottom</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0, 0</td>
<td>1, 0</td>
</tr>
<tr>
<td></td>
<td>0, -1</td>
<td>-1, 3</td>
</tr>
</tbody>
</table>

8. but if allow mixed strategies (and people only care about expected payoff), then Nash equilibrium will always exist

D. Prisoner’s dilemma
1. 2 prisoners, each may confess (and implicate other) or deny
2. gives payoff matrix

<table>
<thead>
<tr>
<th>Row</th>
<th>Top</th>
<th>Bottom</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>-3, -3</td>
<td>-6, 0</td>
</tr>
<tr>
<td></td>
<td>0, -6</td>
<td>-1, -1</td>
</tr>
</tbody>
</table>

3. note that (confess, confess) is unique dominant strategy equilibrium, but (deny, deny) is Pareto efficient
4. example: cheating in a cartel
5. example: agreeing to get rid of spies
6. problem — no way to communicate and make binding agreements

E. Repeated games
1. if game is repeated with same players, then there may be ways to enforce a better solution to prisoner’s dilemma
2. suppose PD is repeated 10 times and people know it
   a) then backward induction says it is a dominant strategy to cheat every round
3. suppose that PD is repeated an indefinite number of times
   a) then may pay to cooperate
4. Axelrod’s experiment: tit-for-tat

F. Example – enforcing cartel and price wars

G. Sequential game — time of choices matters
H. Example:

<table>
<thead>
<tr>
<th></th>
<th>Column</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Left</td>
<td>Right</td>
</tr>
<tr>
<td>Top</td>
<td>1,9</td>
<td>1,9</td>
</tr>
<tr>
<td>Bottom</td>
<td>0,0</td>
<td>2,1</td>
</tr>
</tbody>
</table>

1. (Top, Left) and (Bottom, Right) are both Nash equilibria
2. but in extensive form (Top, Left) is not reasonable. Figure 27.6.

3. to solve game, start at end and work backward
4. (Top, Left) is not an equilibrium, since the choice of Top is not a credible choice

I. Example: entry deterrence
1. stay out and fight
2. excess capacity to prevent entry — change payoffs
3. see Figure 27.7.
4. strategic inefficiency
Figure 27.7
Exchange

A. Partial equilibrium — theory of single market

B. General equilibrium — interactions among many markets
   1. complements and substitutes
   2. prices affect income . . . but income affects prices

C. We do pure exchange first, then production

D. Edgeworth box
   1. Figure 28.1.

2. allocation
3. feasible allocation
4. consumption bundles
5. initial endowment
6. final allocation

E. Trade
   1. move to Pareto preferred point
   2. keep going until no more mutually preferred trades
F. Pareto efficient allocations
1. where trade stops — no mutual improvement possible
2. Pareto efficient — no way to make both people better off
3. indifference curves must be tangent
4. Pareto set, or contract curve — locus of all PE points

G. Market trade
1. specific way to trade — using price system
2. gross demands and net demands; Figure 28.3.

3. market equilibrium — where supply equals demand
4. see Figure 28.4.

H. Algebra
1. only one of the markets needs to clear
2. Walras’s law: if each individual satisfies his or her budget constraint, then the market as a whole must satisfy its budget constraint
3. existence of equilibrium?
I. Efficiency
1. does the market exhaust all the gains from trade?
2. is the market outcome efficient?
3. First Theorem of Welfare Economics — yes
4. is any efficient allocation a market equilibrium?
5. Second Theorem of Welfare Economics — yes, if things are appropriately convex
6. see Figure 28.8.

J. Meaning of First Welfare Theorem
1. implicit assumptions — no externalities
2. competitive behavior
3. existence
4. shows that there is a general mechanism that will achieve efficient outcomes
5. can decentralize decisions

K. Meaning of Second Welfare Theorem
1. prices play allocative and distributive role
2. use market for allocative role and income redistribution for distributive role
3. but problems in production economy
   a) how to measure endowments?
   b) how to redistribute endowments?
Figure 28.8
Production

A. Want to study production in a general equilibrium context
   1. two-good model is somewhat artificial
   2. but necessary for a graphical treatment

B. Robinson Crusoe economy
   1. Robinson is both consumer and producer
   2. consumes leisure and coconuts
   3. can make leisure-consumption choice directly as in Figure 29.1.

   ![Figure 29.1]

   4. or can make it indirectly via the market

C. Crusoe, Inc. — the firm’s choices
   1. firm looks at prices and chooses a profit-maximizing plan
   2. generates some profits $\pi^*$. See Figure 29.2.

D. Robinson the consumer
   1. Robinson collects profits as nonlabor income
   2. looks at price and wage and decides how much to work
   3. chooses optimal consumption point. See Figure 29.3.

E. In equilibrium, demand equals supply
   1. demand for labor equals supply of labor
   2. demand for consumption equals supply of consumption
C

Profit = \pi^*

COCONUTS

Isoprofit line

Production function

Figure 29.2

Figure 29.3

F. Decentralization
1. each “agent” in the economy only has to look at the prices and make his own decisions
2. the consumer doesn’t have to know anything about the production problem
3. the producer doesn’t have to know anything about the consumer’s problem
4. all information is conveyed in prices
5. in a one-person economy, this is silly
6. but in a many-person economy, there can be great savings

G. Different kinds of technologies
1. constant returns to scale — zero profits
2. decreasing returns to scale — positive profits
3. increasing returns to scale — competitive markets don’t work.
   Natural monopoly problem

H. Welfare theorems
1. First welfare theorem — competitive markets are Pareto efficient
2. Second welfare theorem — any Pareto efficient outcome can be achieved by competitive markets

I. Production possibilities
1. if there is more than one good, we can illustrate the production set. Figure 29.7.
2. if there is more than one way to produce output, producers can exploit comparative advantage. Figure 29.8.

3. production possibilities and the Edgeworth box. Figure 29.9.
Slope = $MRT$

Pareto set

Equilibrium production

Slope = $MRS$

Production possibilities set

Equilibrium consumption

Figure 29.9
Welfare

A. Incorporate distributional considerations into the analysis

B. Need some way to compare individual preferences or utilities

C. Aggregation of preferences
   1. majority voting
   2. paradox of voting; see Table 30.1
   3. rank order voting
   4. dependence of irrelevant alternatives; see Table 30.2

D. Arrow’s impossibility theorem

E. Social welfare functions
   1. add together utilities in some way
   2. classical utilitarian: $\sum_{i=1}^{n} u_i$
   3. weighted sum of utilities: $\sum_{i=1}^{n} a_i u_i$
   4. minimax: $\min \{u_1, \ldots, u_n\}$

F. Maximizing welfare
   1. every welfare maximum is Pareto efficient. Figure 30.1.
      
      ![Figure 30.1](image_url)

      2. every Pareto efficient allocation is welfare maximum (if utility possibilities set is convex)
G. Fair allocations

1. generalized the idea of symmetric treatment
2. if $u_i(x_j) > u_i(x_i)$, then we say that $i$ envies $j$
3. typically will be possible to find allocations that are envy-free and efficient
4. proof: start out with equal division and let people trade using a competitive market
5. end up with equal incomes; if someone envies someone else, then they couldn’t have purchased the best bundle they could afford
Externalities

A. Consumption externality occurs when an agent cares directly about another agent’s consumption or production of some good
1. playing loud music
2. smoking a cheap cigar

B. Production externality occurs when a firm’s production function depends on choices of another firm or consumer
1. apple orchard and honeybees
2. pollution

C. Example: smokers and nonsmokers
1. two roommates who consume smoke and money; one likes smoke, the other doesn’t
2. depict preferences
3. depict endowment
   a) each has $100
   b) but what is initial endowment of smoke?
   c) endowment depends on legal system — just like rights to private property
d) right to clean air
e) right to smoke
f) Pareto efficient amounts of smoke and money
g) contract curve; how to trade
h) Figure 31.1.
i) price mechanism generates a “price of smoke”
j) problems arise because property rights are poorly determined.
4. under some conditions, the amount of smoke is independent of the assignment of property rights. Figure 31.2.

D. Production externalities
1. $S$, a steel firm and $F$, a fishery
2. steel: $\max_s p_s s - c_s(s, x)$
3. fishery: $\max_f p_f f - c_f(f, x)$
4. FOC for steel mill:
   \[ p_s = \frac{\partial c_s}{\partial s} \]
   \[ 0 = \frac{\partial c_s}{\partial x} \]
5. FOC for fishery:
   \[ p_f = \frac{\partial c_f(f, x)}{\partial f} \]
Possible endowment $E$
Possible equilibrium $X$
Possible equilibrium $X'$
Possible endowment $E'$
A's indifference curves
B's indifference curves
Pareto efficient allocations

Figure 31.1

Figure 31.2

E. Efficient solution
1. merge and maximize joint profits
2. **internalize** the externality
3. 
   \[
   \max_{s,f} p_s s + p_f f - c_s(s, x) - c_f(f, x)
   \]
4. get \( p_s = \frac{\partial c_s}{\partial s} \), \( p_f = \frac{\partial c_f}{\partial f} \) and 
   \[
   0 = \frac{\partial c_s}{\partial x} + \frac{\partial c_f}{\partial x} = \text{marginal social cost}
   \]
5. joint firm takes interaction into account
6. private costs and social costs
7. how to get firms to recognize social cost
   a) Pigouvian tax — set price of pollution to equal social cost
   b) market pollution rights
   c) assign property rights and let firms bargain over amount of pollution
8. market solution to externalities
   a) either firm has incentive to buy out the other and internalize the externality
   b) since profits from coordination are greater than profits without
   c) sometimes works with consumption externalities
Information Technology

A. Systems competition
   1. info tech components are *complements*
   2. worry about complementers as much as competitors

B. CPU and OS as complements
   1. price of system is $p_1 + p_2$ so demand is $D(p_1 + p_2)$
   2. each setting price alone ignores spillover effect
   3. merging internalizes this externality
   4. other ways to solve the problem of complements
      a) negotiate
      b) revenue sharing
      c) “commoditize the complement”

C. Lock-in
   1. cost of switching
   2. when very large, we have lock-in
   3. e.g., switching ISPs

D. Example of switching ISPs
   1. $c =$ cost of providing service
   2. $p =$ price of service
   3. if no switching costs, $p = c$
   4. now add switching cost of $s$
   5. allow seller to discount first period by $d$
      a) consumer switches if
         \[(p - d) + \frac{p}{r} + s > p + \frac{p}{r}\]
      b) implies $d = s$, which means supplier covers switching costs
   6. competition forces profit to zero
      \[(p - s) - c + \frac{p - c}{r} = 0\]
      a) implies
         \[p = c + \frac{r}{1 + r}s\]
      b) interpretation: ISP invests in discount, earns back premium over cost in subsequent periods

E. Network externalities occur when the value of a good to one consumer depends on how many other consumers purchase it.
   1. examples: fax machines, modems, Internet connections, . . .
F. Model: think of 1,000 people who have willingness to pay of 
v = 1, 2, 3, . . . , 1000.
1. so number of people with willingness to pay greater or equal
to $p$ is $1000 - p$.
2. this is, in fact, the demand curve for the good

G. But now suppose that the value of a fax machine is $vn$, where 
n is the number of people who purchase a fax machine
1. if the price is $p$, then the marginal person satisfies
   \[ p = \hat{v}n \]
2. everyone with value greater than this person buys the fax
   machine, so
   \[ n = 1000 - \hat{v} \]
3. putting these two equations together gives us
   \[ p = n(1000 - n) \]
4. note the peculiar shape of this demand curve!

H. Suppose the fax machines are produced at a constant marginal
   cost of $c$
1. there will then be 3 levels of output where demand equals
   supply
2. note that the middle equilibrium is unstable; if costs decrease
   over time, then system may reach “critical mass”
3. examples: Adobe, Internet

I. Rights management
1. offering more liberal terms and conditions increases value,
   decreases sales
2. baseline case
   a) $y = \text{amount consumed}$
   b) $p(y) = \text{inverse demand}$
   c) $\max_y p(y)y$
3. more liberal terms and conditions
   a) $Y = \beta y$ with $\beta > 1$
   b) $P(Y) = \alpha p(Y)$ with $\alpha > 1$
   c) $\max_Y \alpha p(Y) \frac{Y}{\beta}$
   d) $\max_Y \frac{\alpha}{\beta} p(Y) Y$
4. conclusions
   a) same amount consumed
   b) less produced
   c) profits go up if $\alpha > \beta$, down if inequality reversed

J. Sharing intellectual property
1. examples of sharing
2. monopoly profit maximization: $p(y)y - cy - F$ gives output $\hat{y}$.
3. What if good is shared among $k$ users? If $y$ copies produced, $x = kx$ copies used, so marginal wtp is $p(x)$. Inconvenience of sharing gives us marginal wtp of $p(x) - t$.
4. What about demand by group? It is $k[p(ky) - t]$.
5. willingness to pay goes up due to $k$ in front, down due to $k$ in argument

K. profit maximization

$$\max_y k[p(ky) - t]y - cy - F$$

L. rearrange:

$$\max_x p(x)x - \left(\frac{c}{k} + t\right)x - F.$$

M. Marginal cost in this problem is $(c/k + t)$. How does this compare to marginal cost in original problem?
1. Profits will be larger when rental is possible when

$$\frac{c}{k} + t < c$$

or,

$$\left(\frac{k}{k+1}\right)t < c.$$ 

a) If $k$ is large, this reduces to $t < c$.
2. Interpretation: is it cheaper to produce an extra copy or have an existing copy shared among more consumers?
Public Goods

A. Public goods involve a particular kind of externality — where the same amount of the good has to be available to everyone.

B. Examples: national defense, street lights, roads, etc. — same amount must be provided to all.

C. But people can value the public good in different ways.

D. Private goods
   1. each person consumes different amount, but values it the same (at the margin).

E. Public goods
   1. each person consumes the same amount, but values it differently.

F. Two questions about public goods
   1. what is the optimal amount of a public good?
   2. how well do various social institutions work in providing the optimal amount of a public good?

G. Example: a TV for two roommates. Roommate $i$ will contribute $g_i \geq 0$ towards the purchase. TV will be purchased if $g_1 + g_2 \geq C$.
   1. consider the reservation prices $r_1$ and $r_2$. These measure maximum willingness-to-pay for TV by each person.
   2. suppose that we can find $(g_1, g_2)$ such that $r_1 \geq g_1$, $r_2 \geq g_2$ and $g_1 + g_2 \geq C$.
   3. then clearly it is a good idea to provide the TV.
   4. so, if $r_1 + r_2 \geq C$, then we can find $g_1$ and $g_2$ that cover costs and should provide the TV.
   5. if $r_1 + r_2 < C$, then shouldn’t provide the TV.
   6. condition for efficiency is that the sum of the willingnesses to pay must exceed the cost of provision.
   7. in case of divisible good (e.g., how much to spend on TV), the optimum occurs when the sum of the marginal willingness-to-pay equals marginal cost.
      a) if sum of $MRS$s exceeds marginal cost, can make everyone better off by increasing the amount of public good.
      b) if sum of $MRS$s is less than marginal cost, then should reduce the amount of the public good.
H. Example of divisible good
1. two people each contributes $g_i$ to a TV. Person $i$ gets utility
   $$u_i(g_1 + g_2) - g_i.$$ 
2. efficient allocation maximizes sum of utilities:
   $$\max u_1(g_1 + g_2) + u_2(g_1 + g_2) - g_1 - g_2.$$ 
3. FOC:
   $$u'_1(g_1^* + g_2^*) + u'_2(g_1^* + g_2^*) = 1.$$ 
4. this determines the optimal amount of the public good:
   $$G^* = g_1^* + g_2^*.$$ 
5. if there were $n$ people, the condition would be
   $$\sum_{i=1}^{n} u'_i(G^*) = 1.$$ 

I. Consider various social institutions to provide the public good.
1. voluntary contributions
   a) person 1 will contribute until $u'_1(g_1 + g_2) = 1$.
   b) person 2 will contribute until $u'_2(g_1 + g_2) = 1$.
   c) person who has higher willingness to pay will contribute the entire amount.
   d) other person free rides — contributes zero.
2. majority voting
   a) assume that there are $n > 2$ people.
   b) suppose that each person pays $1/n$ of the public good if it is provided.
   c) if $G$ units of the public good are provided, then person $i$ gets benefit
      $$u_i(G) - \frac{1}{n} G.$$ 
   d) person $i$ will vote for an increase in the amount of the public good if
      $$u'_i(G) > \frac{1}{n}.$$ 
   e) if a majority of the people vote for an increase in the public good, then we get a small increase.
   f) so the amount of the public good is determined by the condition that the median voter is happy with the current amount.
      1) median voter means half the voters want more, half the voters want less.
2) if $m$ is the median voter, then want

$$u'_m(G) = \frac{1}{n}.$$ 

g) in general, this won’t be the optimal amount of the public good.

h) think of case where some voters really want a lot more of the public good and would be willing to compensate those who don’t want more.

i) voting doesn’t take into account *intensity* of preference.

3. Clarke-Groves tax

a) in order to get an efficient amount of the public good, each person must face the social costs of his decision.

b) there are ways of “bidding” for public good that do this.
Asymmetric Information

A. Up until now, we have assumed complete information — consumers and firms know the quality of the goods they buy and sell.

B. But in real life, information may often be incomplete.

C. Then people have to infer quality from price or other signals.

D. Firms may supply such signals intentionally or inadvertently:
   1. used cars — why are you selling it?
   2. warranties — signal of quality.

E. Model of used-car market:
   1. 50 lemons for sale, 50 plums.
   2. buyers willing to pay $2,400 for plum, $1,200 for lemon.
   3. sellers will sell plum for $2,000 and lemon for $1,000.

   4. Full information solution:
      a) plum sells for price between $2,400 and $2,000.
      b) lemon sells for price between $1,200 and $1,000.

   5. Incomplete information solution:
      a) can’t tell if car is a plum or a lemon.
      b) estimate quality by looking at average quality of cars on the market.
      c) suppose all cars were offered for sale.
      d) then the willingness to pay for a car would be
         \[ \frac{1}{2} \times 2,400 + \frac{1}{2} \times 1,200 = 1,800. \]
      e) at this price, the owners of plums wouldn’t sell.
      f) only owners of lemons would sell.
      g) but then the maximum that buyers would be willing to pay would be $1,200!
      h) only equilibrium is for lemons to get offered on market, and price to be between $1,000 and $1,200.
      i) the bad cars have “driven out” the good cars.
      j) there is an externality between the good and bad cars.

F. Quality choice:
   1. in the lemons model, quality is exogenous; what if quality is endogenous?
   2. Umbrella market:
      a) consumers are willing to pay $14 for a high-quality umbrella, $8 for a low-quality umbrella.
b) if a fraction \( q \) are high quality, willing to pay

\[
p = 14q + 8(1 - q).
\]

c) suppose it costs $11.50 to produce high quality and $11 to produce low quality
d) then if there are many firms, and each thinks that it will have a negligible effect on the price, each will choose to produce a low-quality umbrella
e) but the amount that people are willing to pay for a low-quality umbrella ($8) exceeds the cost of production ($11)
f) the possibility of production of a low-quality good has destroyed the market!

3. Adverse selection
   a) consider insurance market
   b) people who need insurance the most are more likely to buy it
c) rates based on average experience over population will not necessarily cover costs
d) high-risk consumers can drive out low-risk consumers
e) mandatory insurance can make people better off on average

G. Signaling
   1. we have seen that when quality in market is mixed, the bad quality can drive out the good
   2. incentive for firms to identify high-quality goods by sending a signal to the consumers
   3. example: a warranty
   4. high-quality producers can afford to offer a warranty, low-quality producers can’t
   5. in equilibrium a warranty can differentiate the two qualities
   6. example—signaling by educational choice
      a) two kinds of workers, able and unable
      b) able have \( MP \) of \( a_2 \), unable have an \( MP \) of \( a_1 \), and \( a_2 > a_1 \).
      c) if firm can observe quality of worker, each type gets paid its \( MP \)
      d) if can’t observe quality, must pay wage equal to average of \( MP \)s
      e) suppose that there is some signal that they can acquire that will indicate which type they are
      f) for example, suppose that workers can choose education level
      g) more able workers have cheaper costs of acquiring education (not necessarily dollar costs)
h) then can have an equilibrium where able workers acquire education, to distinguish themselves from unable workers  
i) *even though* the education doesn’t change their $MP$  
j) socially wasteful investment — only use is to distinguish one group from another