Compositional Semantics

Jacob Andreas
Problem 1

Each of the three girls has a platypus.

Each of the three girls climbed the mountain.

How many platypuses?

How many mountains?
Problem 1

Each of the three girls has a platypus.
Each of the three girls climbed the mountain.
There are 128 cities in South Carolina.

<table>
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<th>name</th>
<th>type</th>
<th>coastal</th>
</tr>
</thead>
<tbody>
<tr>
<td>Columbia</td>
<td>city</td>
<td>no</td>
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<tr>
<td>Cooper</td>
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<td>yes</td>
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<td>Charleston</td>
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</tbody>
</table>
Barack Obama was the 44th President of the United States. *Obama was born on August 4 in Honolulu, Hawaii.* In late August 1961, Obama's mother moved with him to the University of Washington in Seattle for a year…

Is Barack Obama from the United States?
It’s not enough to have structured representations of syntax: We also need structured representations of **meaning**.
It’s not enough to have structured representations of syntax: We also need structured representations of meaning.

Today:
How do we get from language to meaning?
PART I
What is meaning?
 Meaning in formal languages

\[ a + b = 17 \]
Meaning in formal languages

\[ a + b = 17 \]
Meaning in formal languages

\[ a + b = 17 \]

\[ a = ? \]

\[ b = ? \]
Meanings are sets of valid assignments

\[ a + b = 17 \]

- \( \{a=0, b=0\} \)
- \( \{a=3, b=10\} \)
- \( \{a=5, b=12\} \)
- \( \{a=17, b=0\} \)
- \( \{a=10, b=7\} \)
- \( \{a=5, b=5\} \)
Meanings are sets of valid assignments

\[ a + b = 17 \]

\[ \begin{align*}
\{a=0, b=0\} & \times \\
\{a=3, b=10\} & \times \\
\{a=5, b=12\} & \checkmark \\
\{a=17, b=0\} & \checkmark \\
\{a=10, b=7\} & \checkmark \\
\{a=5, b=5\} & \times
\end{align*} \]
Meanings are sets of valid assignments

\[ a + 3 = 20 - b \]

- \{a=0, b=0\} \times
- \{a=3, b=10\} \times
- \{a=5, b=12\} \checkmark
- \{a=17, b=0\} \checkmark
- \{a=10, b=7\} \checkmark
- \{a=5, b=5\} \times
Meanings are *functions* that judge validity

\[
\begin{align*}
\text{Meanings are } & \text{*functions* that judge validity} \\
\{a=5, b=12\} & \rightarrow [a + b = 17] \\
\end{align*}
\]
Meanings are *functions* that judge validity

\[ a + b = 17 \]

\{a=3, b=10\} ×
Lessons from math

\[ a + b = 17 \]

The meaning of a statement is the set of possible worlds consistent with that statement.

Here, a “possible world” is an assignment of values to variables.

\{a=3, b=10\}
Pat likes Sal.
### Representing possible worlds

<table>
<thead>
<tr>
<th>Individuals</th>
<th>Pat</th>
<th>Sal</th>
</tr>
</thead>
<tbody>
<tr>
<td>Properties</td>
<td>whale ←</td>
<td>sad ←</td>
</tr>
<tr>
<td>Relations</td>
<td>loves ➔</td>
<td>contains ➔</td>
</tr>
</tbody>
</table>
Example world

Pat

Sal

Sam

Lou
Example world

- Worried is related to Pat.
- Loves and likes connect Sam to Pat.
- Likes connects Sal to Pat and Sam.
- Contains connects Lou to Sam.
- Shark connects Lou to Sam.
- Happy connects Sal to Lou.
Different example world

- Pat loves Sal
- Sal loves Sam
- Sam loves Pat
- Lou is sad
Representing possible worlds

**Individuals**
- **Pat**
- **Sal**

**Properties**
- whale={Lou}, sad={Pat, Sal}

**Relations**
- likes={((Pat, Sal), (Sal, Sam))}
Pat likes Sal.
Lou is a shark.
Sam is inside Lou, a shark.
The meaning of a sentence is the set of possible worlds it picks out.
PART II

How is meaning constructed?
Explicit representation is too hard

*Pat likes Sal.*
Meanings as functions
Meanings as logical statements

\[ \text{[Pat likes Sal]} \]

likes(Pat, Sal)
Pat likes Sal
likes(Pat, Sal)
Meanings as logical statements

Lou is a shark

shark(Lou)
Meanings as logical statements

Sam is inside Lou, a shark
Sam is inside Lou, a shark

\[
\text{shark}(\text{Lou}) \land \text{contains}(\text{Lou}, \text{Sam})
\]
Meanings as logical statements

Nobody likes Lou
Meanings as logical statements

Nobody likes Lou

∀x. ¬likes(x, Lou)
Meanings as logical statements

Everyone who knows Sal is happy
Meanings as logical statements

Everyone who knows Sal is happy

\( \forall x. \, \text{knows}(x, \text{Sal}) \rightarrow \text{happy}(x) \)
KEY IDEA

Collections of possible worlds can be compactly represented with logical forms.
Compositionality of meaning

Pat likes Sal

Lou is a shark

Sam is inside Lou, a shark

Nobody likes Lou

likes(Pat, Sal)

shark(Lou)

shark(Lou) ∧ contains(Lou, Sam)

∀x.¬likes(x, Lou)
Pat likes Sal

Lou is a shark

Sam is inside Lou, a shark

Nobody likes Lou

Compositionality of meaning
Compositionality of meaning

Pat likes Sal: \text{likes(Pat, Sal)}

Lou is a shark: \text{shark(Lou)}

Sam is inside Lou, a shark: \text{shark(Lou)} \land \text{contains(Lou, Sam)}

Nobody likes Lou: \forall x. \neg \text{likes(x, Lou)}
Compositionality of meaning

A Sal le gusta Pat

Lou es un tiburón

Sam está dentro de Lou, un tiburón

A nadie le gusta Lou

\[ \text{likes}(Pat, Sal) \]

\[ \text{shark}(Lou) \]

\[ \text{shark}(Lou) \land \text{contains}(Lou, Sam) \]

\[ \forall x. \neg \text{likes}(x, Lou) \]
Compositionality of meaning

\[
\begin{align*}
a_{12} & \ b_5 \ c_{67} \ a_8 & \text{likes}(\text{Pat, Sal}) \\
a_{12} & \ b_5 \ c_0 \ a_0 & \text{shark}(\text{Lou}) \\
a_{12} & \ b_{16} \ c_{12} \ c_{12} & \text{shark}(\text{Lou}) \wedge \text{contains}(\text{Lou, Sam}) \\
a_{53} & & \forall x. \neg \text{likes}(x, \text{Lou})
\end{align*}
\]
KEY IDEA

Pieces of logical forms correspond to pieces of language
Building a lexicon

Sam is inside Lou, a shark \[\text{shark}(\text{Lou}) \land \text{contains}(\text{Lou}, \text{Sam})\]

Pat: Pat
Sal: Sal
Sam: Sam
Lou: Lou
Building a lexicon

Sam is inside Lou, a shark, \( \text{shark}(\text{Lou}) \land \text{contains}(\text{Lou}, \text{Sam}) \)

Pat: Pat
Sal: Sal
Sam: Sam
Lou: Lou
Building a lexicon

Sam is inside Lou, a shark \( \text{shark}(\text{Lou}) \land \text{contains}(\text{Lou}, \text{Sam}) \)

Pat: Pat \hspace{1cm} \text{shark: } \lambda x.\text{shark}(x)\hspace{1cm} \\
Sal: Sal \\
Sam: Sam \\
Lou: Lou
Building a lexicon

\[\text{Sam is inside Lou, a shark} \quad \text{shark}(\text{Lou}) \land \text{contains(Lou, Sam)}\]

\begin{align*}
\text{Pat: Pat} & \quad \text{shark: } \lambda x.\text{shark}(x) \\
\text{Sal: Sal} & \quad \text{likes: } \lambda yx.\text{likes}(x, y) \\
\text{Sam: Sam} & \quad \text{nobody: } \lambda f.\forall x.\neg f(x) \\
\text{Lou: Lou} & \quad \ldots
\end{align*}
What do we do now?

Pat          sent          Lou          a            letter

\[ \lambda xy.\text{sent}(x,y,z) \quad \lambda f.\text{Ax.f}(x) \quad \lambda x.\text{letter}(x) \]
What do we do now?

What do we do now?

What do we do now?
What do we do now?

[Diagram showing logical expressions involving Pat, Lou, and a letter, with lambda expressions for sent and letter]
What do we do now?

letter($\lambda f. Ax. f(x)$)?
What do we do now?

- Pat: $\lambda y z x . \text{sent}(x, y, z)$
- Lou: $\lambda f . \text{Ax.f}(x)$
- $\lambda x . \text{letter}(x)$
What do we do now?

Pat          sent          Lou            urgently

λyzx.sent(x,y,z)
Semantic types

\[ \text{Pat} \quad \text{sent} \quad \text{Lou} \quad \text{a} \quad \text{letter} \]

\[ \lambda yzx.\text{sent}(x,y,z) \quad \lambda f.\text{Ax.f}(x) \quad \lambda x.\text{letter}(x) \]

Object \downarrow \text{Bool} \quad \text{Object} \downarrow \text{Bool} \quad \text{Object} \downarrow \text{Bool}
Semantic types & syntax

Pat sent Lou a letter

\( \lambda yzx.\text{sent}(x,y,z) \) \( \lambda f.\text{Ax.f}(x) \) \( \lambda x.\text{letter}(x) \)

NP, NP, NP

NP, NP, NP

NP

NP

NP

NP

NP

NP

NP
**Semantic types & syntax**

<p>| | | | | | |</p>
<table>
<thead>
<tr>
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<td><em>Pat</em></td>
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<td><em>Lou</em></td>
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<td></td>
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<tr>
<td>Pat</td>
<td>( \lambda yzx.\text{sent}(x,y,z) )</td>
<td>Lou</td>
<td>( \lambda f.\text{Ax}.f(x) )</td>
<td>( \lambda x.\text{letter}(x) )</td>
<td></td>
</tr>
<tr>
<td>NP</td>
<td>( ((S</td>
<td>NP)</td>
<td>NP)</td>
<td>NP )</td>
<td>NP</td>
</tr>
</tbody>
</table>
## Categorial grammar

<table>
<thead>
<tr>
<th></th>
<th>\textit{Pat}</th>
<th>\textit{sent}</th>
<th>\textit{Lou}</th>
<th>a</th>
<th>\textit{letter}</th>
</tr>
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<tr>
<td>NP</td>
<td>Pat ( \lambda \text{yzx}. \text{sent}(x,y,z) )</td>
<td>\textit{sent}</td>
<td>Lou ( \lambda f.Ax.f(x) )</td>
<td>a ( \lambda x. \text{letter}(x) )</td>
<td>letter</td>
</tr>
<tr>
<td></td>
<td>NP ((S/NP)/NP)/NP</td>
<td></td>
<td>NP | NP/(S/NP)</td>
<td></td>
<td>S/NP</td>
</tr>
</tbody>
</table>
Parsing with a categorial grammar

\[
\begin{align*}
&\text{Pat} \quad \text{sent} \quad \text{Lou} \quad a \quad \text{letter} \\
&\lambda \text{lyzx.} \text{sent}(x,y,z) \quad \lambda f.\text{Ax.}f(x) \quad \lambda x.\text{letter}(x) \\
&\text{NP} \quad ((S\text{NP})/\text{NP})/\text{NP} \quad \text{NP} \quad \text{NP}/(S/\text{NP}) \quad S/\text{NP} \\
&\underline{Ax.\text{letter}(x)} \quad \text{NP}
\end{align*}
\]
Parsing with a categorial grammar

\[
\begin{array}{cccccc}
\text{Pat} & \text{sent} & \text{Lou} & \text{a} & \text{letter} \\
\lambda y z x . \text{sent}(x, y, z) & \lambda f. A x . f(x) & \lambda x . \text{letter}(x) \\
((S\backslash N P)/N P)/N P & N P/(S\backslash N P) & S/ N P \\
\lambda z x . \text{sent}(x, \text{Lou}, z) & \text{Ax.letter}(x) & N P \\
(S\backslash N P)/N P & \\
\end{array}
\]
Parsing with a categorial grammar

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<td>$\lambda yzx.\text{sent}(x,y,z)$</td>
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<td>$\lambda x.\text{letter}(x)$</td>
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<td>NP</td>
<td>$((S\backslash NP)/NP)/NP$</td>
<td>NP</td>
<td>$NP/(S/\text{NP})$</td>
<td>$S/\text{NP}$</td>
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$\lambda zx.\text{sent}(x,\text{Lou},z)$

$(S\backslash NP)/NP$

$\lambda x.\text{sent}(x,\text{Lou},Ax.\text{letter}(x))$  $S/\text{NP}$
Parsing with a categorial grammar

\[
\begin{align*}
\text{Pat} & \quad \text{sent} \quad \text{Lou} \quad \text{a} \quad \text{letter} \\
\lambda y z x. \text{sent}(x,y,z) & \quad \lambda f. A x. f(x) \quad \lambda x. \text{letter}(x) \\
((S\NP)/\NP)/\NP & \quad \NP/(S/\NP) \quad S/\NP \\
\text{NP} & \quad \NP \\
\text{NP} & \quad \text{NP} \\
\lambda z x. \text{sent}(x,\text{Lou},z) & \quad \text{Ax. letter}(x) \\
(S\NP)/\NP & \quad \NP \\
\lambda x. \text{sent}(x,\text{Lou},\text{Ax. letter}(x)) & \quad S\NP \\
\text{sent}(\text{Pat},\text{Lou},\text{Ax. letter}(x)) & \quad S
\end{align*}
\]
Semantics → Synax!

Pat sent Lou a letter

___________________   ____________

___________________

___________________
Pat sent Lou a letter
**Key Idea**

Types in logic correspond to grammatical categories in language.
Problem 1

Each of the three girls has a platypus.

Each of the three girls climbed the mountain.

\[ \forall x. \text{girl}(x) \rightarrow \exists y. \text{platypus}(y) \land \text{has}(x, y) \]

\[ \exists y. \text{mountain}(y) \land \forall x. \text{girl}(x) \rightarrow \text{climbed}(x, y) \]
Problem 2

There are 128 cities in South Carolina

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Problem 2

There are 128 cities in South Carolina

\[
\text{same}(128, \\
\text{count } x. \text{ city}(x) \land \\
\text{in}(x, \text{SouthCarolina})
\]
Barack Obama was the 44th President of the United States. Obama was born on August 4 in Honolulu, Hawaii. In late August 1961, Obama's mother moved with him to the University of Washington in Seattle for a year…

Is Barack Obama from the United States?
Barack Obama was the 44th President of the United States. Obama was born on August 4 in Honolulu, Hawaii. In late August 1961, Obama's mother moved with him to the University of Washington in Seattle for a year. Is Barack Obama from the United States? Yes.
Problem 3

Barack Obama was the 44th President of the United States. Obama was born on August 4 in Honolulu, Hawaii.

\[
\text{born}(\text{Obama, Hawaii, August 4})
\]

\[
\text{born}(x, y, z) \rightarrow \text{from}(x, y)
\]

\[
\text{from}(x, y) \land \text{in}(y, z) \rightarrow \text{from}(x, z)
\]

\[
\text{in}(\text{Hawaii, United States})
\]

Is Barack Obama from the United States?

Yes?
The meaning of a sentence is the set of possible worlds it picks out.
**Key idea**

Collections of possible worlds can be compactly represented with logical forms.
**KEY IDEA**

Pieces of logical forms correspond to pieces of language
**Key Idea**

Types in logic correspond to grammatical categories in language.
BONUS ROUND
What’s missing?
Q: How do you like my cooking?
Q: How do you like my cooking?
A: It’s extremely interesting.
Q: How do you like my cooking?
A: It’s extremely interesting.

Q: Do you know what time it is?
Q: How do you like my cooking?
A: It’s extremely interesting.

Q: Do you know what time it is?
A: Yes, I do.
Sal might have seen a unicorn.

Pat thinks Sal saw a unicorn.

Pat wants to find a unicorn.
KEY IDEA

Not all meaning is literal!
BONUS ROUND

Historical Notes
ling121: “Logical Semantics”

Ted Briscoe’s lecture notes:
https://www.cl.cam.ac.uk/teaching/1011/L107/semantics.pdf

Mark Steedman, “The Syntactic Process”